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DECOMPOSING COMPLETE GRAPHS INTO SMALL ROSY GRAPHS

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In 1967 A. Rosa introduced some important types of vertex labelings as useful tools for decompositions of complete graphs K_{2n+1} into graphs with n edges. A *labeling* of a graph G with n edges is an injection ρ from the vertex set of G, $V(G)$, into a subset S of the set $\{0, 1, 2, \ldots, 2n\}$ of elements of the additive group Z_{2n+1}. The *length* of an edge xy is defined as $\ell(x, y) = \min\{\rho(x) - \rho(y), \rho(y) - \rho(x))\}$. Notice that the subtraction is performed in Z_{2n+1} and hence both differences are positive. If the set of all lengths of the n edges is equal to $\{1, 2, \ldots, n\}$ and $S \subseteq \{0, 1, \ldots, 2n\}$, then ρ is a *rosy labeling* (called originally ρ-labeling by AR); if $S \subseteq \{0, 1, \ldots, n\}$ instead, then ρ is a *graceful labeling* (called β-labeling by AR). A graceful labeling ρ is said to be an *α-labeling* if there exists a number ρ_0 with the property that for every edge $xy \in G$ with $\rho(x) < \rho(y)$ it holds that $\rho(x) \leq \rho_0 < \rho(y)$. Obviously, G must be bipartite to allow an α-labeling.

A. Rosa proved that if a graph G with n edges has a rosy (or graceful) labeling, then the complete graph K_{2n+1} can by cyclically decomposed into copies of G. He also showed that if a bipartite graph G with n edges has an α-labeling, then for any positive integer m the complete graph K_{2nm+1} can by cyclically decomposed into copies of G.

We will observe that if a bipartite graph G decomposes K_n and K_m, then it also decomposes K_{nm}. Using this observation, we show that every bipartite graph G with n edges and a rosy labeling decomposes $K_{(2n+1)^k}$ for any positive integer k.