



Invited lecture

ON 1, 2 CONJECTURE

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A *k*-total-weighting of a simple graph G is an assignment of an integer weight, $w(e), w(v) \in \{1, \dots, k\}$ to each edge e and each vertex v of G . The *k*-total-weighting is *neighbour-distinguishing* if for every edge uv , $w(u) + \sum_{e \ni u} w(e) \neq w(v) + \sum_{e \ni v} w(e)$. If it exists, we say that G *permits* a neighbour-distinguishing *k*-total-weighting. The smallest k for which G permits a neighbour-distinguishing *k*-total-weighting we denote by $\tau(G)$.

Similar parameter, but in the case of an *edge* (not total) weighting was introduced and studied in by Karoński, Łuczak and Thomason. They asked if each, except for a single edge, simple connected graph permits a *neighbour-distinguishing 3-edge-weighting*, and showed that this statement holds e.g. for the 3-colourable graphs. It is also known that each *nice* (not containing a connected component which has only one edge) graph permits a neighbour-distinguishing 16-edge-weighting, hence this parameter is finite.

Note that if a graph permits a neighbour-distinguishing *k*-edge-weighting, then it also permits a neighbour-distinguishing *k*-total-weighting (it is enough to put ones at all vertices), hence we obtain an upper bound $\tau(G) \leq 16$ for all graphs and $\tau(G) \leq 3$ for all graphs if the conjecture by Karoński, Łuczak and Thomason holds.

Therefore, we ask if, maybe, the weights 1 and 2 are enough in the case of a total-weighting. It is also worth mentioning here that our reasonings correspond with the recent study of Bača, Jendrol, Miller and Ryan. They introduced and studied a parameter called a *total vertex irregularity strength*, which is the smallest k for which there exists a *k*-total-weighting such that each vertex of a graph receive a different colour, i.e. $w(u) + \sum_{e \ni u} w(e) \neq w(v) + \sum_{e \ni v} w(e)$ for each (not only neighbouring ones) pair of vertices u, v .

This parameter, as well as the other mentioned in this section, may be viewed as descendants of the well known *irregularity strength* of a graph.

References

- [1] M. Bača, S. Jendrol, M. Miller, J. Ryan, *On irregular total labellings*, Discrete Math., to appear.
- [2] M. Karoński, T. Łuczak, A. Thomason, *Edge weights and vertex colours*, J. Combin. Theory, Ser. B 91 (2004) 151-157.