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ON DOMINATING CLIQUES IN RANDOM GRAPHS

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The phase transition phenomenon, originally observed as a physical effect, used to be often emerged also in discrete structures like random graphs. The most frequent property of random graphs which have been studied with relation to the phase transitions is the connectivity. Our paper deals with another interesting graph problem that is an emerging of dominating cliques in a standard Erdös-Rényi random graph model.

Given a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a *dominating set* of G if each node $v \in V$ is either in S or is adjacent to a node in S. A *clique* in G is a maximal set of mutually adjacent nodes of G, i.e., it is a maximal complete subgraph of G. If a subgraph S induced by a dominating set is a clique in G then S is called a *dominating clique* in G. Given a fixed constant p, $0 < p < 1$, let us consider the standard Erdös-Rényi random graph model denoted by $\mathbb{G}(n, p)$.

Theorem *Let $\mathbb{L}x$ denote $\log_{1/(1-p)} x$. Let r be an order of the clique such that $\lfloor \log_{1/p} n \rfloor \leq r \leq \lceil 2 \log_{1/p} n \rceil$. Let $\delta(n) : \mathbb{N} \to \mathbb{N}$ be an arbitrary slowly increasing function such that $\delta(n) = o(\log n)$ and let $G \in \mathbb{G}(n, p)$ be a random graph. Then it holds:*

1. *If $p > 1/2$, then an r-node clique is dominating in G almost surely;*
2. *If $p \leq (3 - \sqrt{5})/2$, then an r-node clique is not dominating in G almost surely;*
3. *If $(3 - \sqrt{5})/2 < p \leq 1/2$, then an r-node clique:*
   - *is dominating in G almost surely, if $r \geq \mathbb{L}n + \delta(n)$,*
   - *is not dominating in G almost surely, if $r \leq \mathbb{L}n - \delta(n)$,*
   - *is dominating with a finite probability $f(p)$ for a suitable function $f : [0, 1] \to [0, 1]$, if $r = \mathbb{L}n + O(1)$.*

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