DECOMPOSITION OF BIPARTITE GRAPHS INTO CLOSED TRAILS

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Let $\mathrm{Lct}(G)$ be the set of all lengths of closed trails that are present in a graph G with all vertices of even degrees. Further, let $\mathrm{Sct}(G)$ be the set of all sequences of terms of $\mathrm{Lct}(G)$ that add up to $|E(G)|$. A sequence $(t_1, \ldots, t_p) \in \mathrm{Sct}(G)$ is G-realisable if there is a sequence $(T_1, \ldots, T_p)$ of pairwise edge-disjoint closed trails in G such that T_i is of length t_i for $i = 1, \ldots, p$. The graph G is arbitrarily decomposable into closed trails provided all sequences in $\mathrm{Sct}(G)$ are G-realisable. It is proved that if $a \geq 1$ is an odd integer and $M_{a,a}$ is a perfect matching in $K_{a,a}$, then the graph $K_{a,a} - M_{a,a}$ is arbitrarily decomposable into closed trails.