CSGT 2007
Combinatorica o.p.s.
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GRAPHS WITH ODD CYCLES OF LENGTH 5 AND 7 ARE 3-COLOURABLE. GENERALIZATIONS POSSIBLE?

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Let $L(G)$ be the set of odd cycle lengths of a graph G. A well known conjecture of Bollobás and Erdős said that $|L(G)| = k$ implied $\chi(G) \leq 2k+2$; it was later strengthened by Gallai and finally proved by Gyárfás to be true in the following form.

Theorem *If G is a 2-connected graph with minimum degree at least $2k+1$ then $|L(G)| = k \geq 1$ implies $G = K_{2k+2}$.*

We are interested in a particular case when $|L(G)| = 2$ and $3 \notin L(G)$. The theorem then asserts the chromatic number of G is at most 5. But it seems the chromatic number is actually significantly lower than this estimation, namely we believe it is always equal to 3.

We are able to prove this conjecture for a very special case when $L(G) = \{5, 7\}$ so far. Our proof consists of two main stages. The first one is proved for a more general circumstance when $|L(G)| = 2$ and the two odd cycle lengths are two consecutive odd numbers, and it seems to be only a matter of more case analysis to extend it to the desired generality. Unfortunately, the next step of the proof is closely tailored to the considered particular setting; although it could be apparently proved for several other special cases like $L(G) = \{7, 9\}$ for example, we are pretty sure extending of it to the fully general case needs a quite new approach.