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NEW RESULTS ON INTEGRAL GRAPHS

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The notion of integral graphs was first introduced by F. Harary and A. J. Schwenk in 1974. A graph G is called integral if all the zeros of the characteristic polynomial $P(G,x)$ are integers. In general, the problem of characterizing of integral graphs seems to be difficult. That is why some special classes of integral graphs are studied.
Integral trees are one important class of these graphs. We constructed the first known integral trees of diameter 7. It is known that an integral tree of diameter 7 cannot be balanced. We shall code a balanced tree of diameter $2k$ by the tree $T(n_k, n_{k-1}, ..., n_1)$, where n_j $(j = 1, 2, ..., k)$ denotes the number of successors of a vertex at the distance $k - j$ from the centre. Let the tree

$$T(n_k, n_{k-1}, ..., n_1)\Theta T(m_j, m_{j-1}, ..., m_1)$$

be obtained by joining the centre w of $T(n_k, n_{k-1}, ..., n_1)$ and the centre v of $T(m_j, m_{j-1}, ..., m_1)$ with a new edge. This tree is denoted by $T(n_1, n_2, ..., n_{k-1}, n_k; 1; m_j, m_{j-1}, ..., m_1)$. We proved that the trees
$T(49, 480, 270; 1; 270, 420, 64)$
$T(25, 264, 504; 1; 504, 220, 36)$
$T(3136, 5328, 1140; 1; 1140, 5700, 3136)$
$T(625, 4704, 1188; 1; 1188, 4900, 576)$
are integral.
Complete n-partite graphs are another interesting class of integral graphs. Only complete bipartite and complete tripartite integral graphs were known so far. We constructed the first known infinite class of complete 4-partite graphs, which is $K_{117q,261q,352q,495q}$, where $q \in N$.

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