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A CLOSURE CONCEPT IN $K_{1,r}$-FREE GRAPHS

Roman Kužel

A graph G is called $K_{1,r}$*-free* if G has no $K_{1,r}$ as an induced subgraph. A hypergraph H is called k*-uniform* if all its edges have cardinality k. Clearly a 2-uniform hypergraph is a graph in a usual sense. A *line graph* of a hypergraph $H = (V, E)$ is a graph $L(H)$ with vertex set E and with two vertices adjacent in $L(H)$ iff the corresponding edges of H have a nonempty intersection. Let v be a vertex of a graph G, we say that v is *locally connected* if the neighborhood of v in G induces a connected subgraph in G. If a locally connected vertex v induces a non complete subgraph we call the vertex v *eligible*. Let G be a graph and $v \in V(G)$ be its eligible vertex. A graph G_v obtained from a graph G by adding all missing edges into a neighborhood of v is called a *local completion* of G at vertex v. A *closure* of a graph G is a graph $\mathrm{Cl}(G)$ obtained by repeating of an operation of a local completion at eligible vertices until no other eligible vertex remains. A k*-walk* in a graph G is a closed walk visiting each vertex at most k times, where $k \geq 1$ is an integer. Note, that in our notations a k-walk in a graph G need not to span all vertices of G. We can then view a cycle in a graph as an 1-walk. A k*-circumference* of a graph G is a cardinality of a vertex set of a k-walk $W \subset G$ spanning maximum possible number of vertices of G. We denote by $c_k(G)$ the k-circumference of a graph G. We show the following generalization of the well-known Ryjáček's closure concept in $K_{1,3}$-free graphs.

Theorem *Let $r \geq 3$ be an integer and let G be a $K_{1,r}$-free graph. Then*

(i) $c_{r-2}(G) = c_{r-2}(\mathrm{Cl}(G))$,

(ii) $\mathrm{Cl}(\mathrm{G})$ *is unique,*

(iii) there is an $(r-1)$*-uniform hypergraph* H *such that* $\mathrm{Cl}(G) = L(H)$.

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