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Abstracts

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HOMOMORPHISMS OF TRIANGLE GROUPS

Marcel Abas

Triangle group $T(m,n)$, with a presentation $\langle a,b,c|a^2 = b^m = c^n = abc = 1\rangle$, is known as a group of orientation preserving automorphisms of universal map $U(m,n)$. We say that a group homomorphism $\varphi : T(m,n) \to H$ has *injectivity radius at least* r if $\varphi(x) \neq 1_H$ for all non-identity elements $x \in T(m,n)$ such that $|x| \leq r$.

In this contribution we present an upper bound on the order of a homomorphic image of the triangle group $T(m,n)$ with injectivity radius at least r.

STABILITY OF GRAPH PROPERTIES UNDER SUBGRAPH CONTRACTION CLOSURE

Alisher Abdurahmanov

This thesis is a further research in the developing part of the graph theory - the closure concept, which was found very useful in investigation of different hamiltonian properties and the structure of the graph. The work introduces some well known closure concepts, shows the known stability results of various hamiltoninan properties under the so-called claw-free closure introduced by Z. Ryjáček. The research is emphasized on the relatively new closure concept based on contractibility techniques and first introduced by Z. Ryjáček. and Schelp R.H. The main part of the thesis summaries some already known stability results of some hamiltonian properties of graphs, but for the closure based on contractibility techniques, and also gives some new results about 2-factors.

ON SUPER ANTIMAGIC LABELINGS OF TREES

Martin Bača*, Yuqing Lin,
and Francesc A. Muntaner-Batle

A *labeling* of a graph is any map that carries some set of graph elements to numbers (usually to the positive integers). Magic labelings are one-to-one maps onto the appropriate set of consecutive integers starting from 1, with some kind of "sum" property. An *edge-magic total labeling* on a graph with p vertices and q edges is defined as a one-to-one map taking the vertices and edges onto the integers $1, 2, \ldots, p+q$ with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge. If the sums of the labels on the edges and the labels of their endpoints form an arithmetic progression starting from a and having common difference d then the labeling is called (a, d)-*edge-antimagic total*. Such a labeling is called *super* if the smallest possible labels appear on the vertices

We will present super (a, d)-edge-antimagic total properties for a special class of trees called *path-like trees*.

THE DISTINGUISHING COLOURING

Kristína Budajová*, Stanislav Jendroľ

Consider a directed graph $G = (V, E)$. Let $\varphi : V \cup E \rightarrow \{1, 2, \ldots, k\}$ be an assignment of colours from the set $\{1, 2, \ldots, k\}$ to the vertices and edges of G. Such a colouring is called *the total k-colouring*. Assign each edge $e = (x, y)$ the ordered triple of integers $(\varphi(x), \varphi(e), \varphi(y))$, where $\varphi(x)$ is colour assigned to the initial vertex x of edge e, $\varphi(e)$ is colour of the edge e and $\varphi(y)$ is colour assigned to the terminal vertex of edge e. The total k-colouring of a directed graph G is called *the edge-irregular k-colouring* if for any two different edges $e = (x, y)$ and $f = (u, v)$ the associated ordered triplets $(\varphi(x), \varphi(e), \varphi(y))$ and $(\varphi(u), \varphi(f), \varphi(v))$ are different. The minimum of k for which the directed graph G has the total edge-irregular k-colouring is called *the total edge-irregular number* of G and is denoted *hit(G)*.

In the paper we will present our recent results concerning the total edge-irregular number of few special families of directed graph.

THE OPEN TRAILS IN DIGRAPHS

SYLWIA CICHACZ*, AGNIESZKA GÖRLICH

The first results on the topic of arbitrarily decomposition of graphs into trails is due to Balister, who proved that if $G := K_n$ for n odd or $G := K_n - I$, where I is a 1-factor in K_n, for n even, then G is arbitrarily decomposable into closed trails ([1]). Horňák and Woźniak ([4]) showed that complete bipartite graphs $K_{a,b}$ for a, b even are also arbitrarily decomposable into closed trails. The notion of an arbitrarily decomposable graphs into closed trails graph were generalized to oriented graphs (see Balister [2] and Cichacz [3]). Balister proved a necessary and sufficient condition for complete digraph to be decomposable into directed closed trails. Whereas Cichacz ([3]) showed that complete directed graphs $\overleftrightarrow{K}_{a,b}$ are arbitrarily decomposable into closed directed trails.

In this article we consider the corresponding question for open trails. We show that complete directed graphs $\overleftrightarrow{K}_n$ and $\overleftrightarrow{K}_{a,b}$ are arbitrarily decomposable into directed open trails.

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THE $(k, n\text{–}k)$-RECONSTRUCTION OF GRAPHS AND DIGRAPHS

Peter Czimmermann

The Reconstruction Conjecture is famous still unsolved problem. The conjecture has been proved for several classes of graphs and graph invariants. It is also known that the conjecture is not true for digraphs - P.Stockmeyer has found an infinite family of non-reconstructible tournaments (Stockmeyer tournaments).

In this talk I would like to present a certain generalization of the graph reconstruction which is called $(k, n-k)$-reconstruction (of graphs and digraphs). It is known that the graphs are $(1, n-1)$-reconstructible if and only if they are reconstructible. Situation is different if we consider digraphs. It is possible to show that the Stockmeyer tournaments do not form a counterexample for $(1, n-1)$-reconstruction of digraphs. Some recent results related to this topic will be discussed.

Invited lecture

ON PACKING IN GRAPHS

Roman Čada

A survey of known results on packing in graphs will be given. Particular attention will be paid to packing paths and cycles in some classes of graphs.

Recent results related to open problems in this area will be presented.

HAMILTONIAN CYCLES IN THE SQUARE OF A GRAPH WITH BLOCK GRAPH HOMEOMORPHIC TO A STAR

Jan Ekstein

Let G be a simple undirected graph. The *square of* G is the graph G^2 with the same vertex set as G, in which two vertices are adjacent if and only if their distance in G is at most 2.

We show that under certain conditions the square of the graph obtained by identifying a vertex in two graphs with hamiltonian square is also hamiltonian. We present (polynomially verifiable) necessary and sufficient conditions for hamiltonicity of the square of a connected graph whose block graph is homeomorphic to a star in which the center corresponds to a cut vertex, and sufficient conditions for hamiltonicity of the square of a connected graph whose block graph is homeomorphic to a star in which the center corresponds to a block.

Invited lecture

HOMOMORPHISMS WITH LOCAL CONSTRAINTS — STRUCTURE AND COMPLEXITY

Jiří Fiala

A graph homomorphism $f : G \to H$ is locally bijective if for every vertex u of G the mapping f acts bijectively between the neighborhood of u and the neighborhood of $f(u)$. We define locally injective and locally surjective homomorphisms analogously.

A degree partition of a graph is an equivalence on its vertices such that equivalent vertices cannot be distinguished by counting their neighbors in any equivalence class. A square matrix that quantitatively describes adjacencies between the classes of a degree partition is called degree matrix.

We express the existence of a locally constrained homomorphism between two graphs as two kinds of binary relations: one on graphs, and another on degree matrices.

These structures have interesting combinatorial properties, e.g. they impose a partial order. In the talk we review some classical as well as some recent results on this subject. We apply them in a study of the computational complexity of the related problems.

DECOMPOSING COMPLETE GRAPHS INTO SMALL ROSY GRAPHS

Dalibor Fronček

In 1967 A. Rosa introduced some important types of vertex labelings as useful tools for decompositions of complete graphs K_{2n+1} into graphs with n edges. A *labeling* of a graph G with n edges is an injection ρ from the vertex set of G, $V(G)$, into a subset S of the set $\{0, 1, 2, \ldots, 2n\}$ of elements of the additive group Z_{2n+1}. The *length* of an edge xy is defined as $\ell(x, y) = \min\{\rho(x) - \rho(y), \rho(y) - \rho(x))\}$. Notice that the subtraction is performed in Z_{2n+1} and hence both differences are positive. If the set of all lengths of the n edges is equal to $\{1, 2, \ldots, n\}$ and $S \subseteq \{0, 1, \ldots, 2n\}$, then ρ is a *rosy labeling* (called originally ρ-labeling by AR); if $S \subseteq \{0, 1, \ldots, n\}$ instead, then ρ is a *graceful labeling* (called β-labeling by AR). A graceful labeling ρ is said to be an α*-labeling* if there exists a number ρ_0 with the property that for every edge $xy \in G$ with $\rho(x) < \rho(y)$ it holds that $\rho(x) \le \rho_0 < \rho(y)$. Obviously, G must be bipartite to allow an α-labeling.

A. Rosa proved that if a graph G with n edges has a rosy (or graceful) labeling, then the complete graph K_{2n+1} can by cyclically decomposed into copies of G. He also showed that if a bipartite graph G with n edges has an α-labeling, then for any positive integer m the complete graph K_{2nm+1} can by cyclically decomposed into copies of G.

We will observe that if a bipartite graph G decomposes K_n and K_m, then it also decomposes K_{nm}. Using this observation, we show that every bipartite graph G with n edges and a rosy labeling decomposes $K_{(2n+1)^k}$ for any positive integer k.

RAINBOW NUMBERS FOR SOME GRAPHS

Izolda Gorgol

A subgraph of an edge-coloured graph is *rainbow* if all of its edges have different colours. For a graph H and a positive integer n, the *anti-Ramsey number* $f(n, H)$ is the maximum number of colours in an edge-colouring of K_n with no rainbow copy of H. The *rainbow number* $rb(n, H)$ is the minimum number of colours such that any edge-colouring of K_n with $rb(n, H)$ number of colours contains a rainbow copy of H. Certainly $rb(n, H) = f(n, H) + 1$. Anti-Ramsey numbers were introduced by Erdős *at al.* [1] and studied in numerous papers.

We present known results on anti-Ramsey numbers and add some new ones.

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EMBEDDING OF GRAPH INTO ITS COMPLEMENT IN TRANSITIVE TOURNAMENT

Agnieszka Görlich*, Monika Pilśniak

Let $\overrightarrow{TT_n}$ be a transitive tournament on n vertices. It is known [1] that for any directed acyclic graph G of order n and of size not greater than $\frac{3}{4}(n-1)$ two directed graphs isomorphic to G are arc disjoint subgraphs of $\overrightarrow{TT_n}$. In this paper we consider a problem of embedding of graphs into its complement in transitive tournament. We show that any directed acyclic graph $\overrightarrow{G}$ of size not greater then $\frac{2}{3}(n-1)$ is embeddable into its complement in $\overrightarrow{TT_n}$. Moreover, this bound is generally the best possible.

References

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CONTRACTIBLE SUBGRAPHS AND FORBIDDEN INDUCED SUBGRAPHS

Ruslan Gumerov

Ryjáček introduced a closure concept in the class of claw-free graphs based on local completion of a locally connected vertex. Brousek et al. studied the stability of some classes of claw-free graphs defined in terms of forbidden subgraphs under the closure and characterized all connected graphs A such that the class of all $(K_{1,3}, A)$-free graphs is stable.

In 2003 Ryjáček and Schelp introduced a closure concept in the class of line graphs based on contractibility of certain subgraphs in the line graph preimage and showed that the closure can be considered as a generalization of the closure concept based on local completion. We study an analogical problem under the generalized closure and prove a characterization theorem.

GOAL-MINIMALLY k-ELONGATED GRAPHS

Štefan Gyürki

Let k be an integer. A 2-edge connected graph G is said to be goal-minimally k-elongated if for every edge $uv \in E(G)$ the distance $d_{G-uv}(x,y) > k$ holds if and only if $\{u,v\} = \{x,y\}$. In particular, if the integer k is equal to the diameter of graph G, we get the class of goal-minimally k-diametric graphs introduced by Kyš in [2] and studied by Gliviak and Plesník in [1] and [3]. We construct some infinite families of goal-minimally k-elongated graphs and explore the goal-minimally properties of cages and symmetric cubic graphs.

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ON THE AZULENOIDS WITH LARGE RING OF 5-GONS

Róbert Hajduk*, František Kardoš, Roman Soták

Deza and Grishukhin studied 3-valent maps $M_n(p,q)$ consisting of a ring of n q-gons whose inner and outer domains are filled by p-gons. They described the conditions on (n,p,q) under which such map may exist and presented several infinite families of them. The open cases are, in particular, $M_n(7,5)$ with $n > 28$, $M_n(5,7)$ with $n \in \{17,18,19\}$ and $M_n(5,q)$ for $q \geq 8$. For the case $M_n(7,5)$, we give all maps with $n \leq 52$. Moreover, we give a construction of maps $M_{48+4k}(7,5)$ for $k \geq 0$ and $M_{42+12k}(7,5)$ for $k \geq 0$. We also construct maps with odd n, $n = 87$. For all these maps we give also their symmetry groups. Among the maps we have found, there are several ones with non-isomorphic inner and outer domains, and, moreover, the one that has only trivial symmetry group.

LIST HOMOMORPHISMS FOR DIGRAPHS

Tomas Feder, Pavol Hell*, Arash Rafiey

I will discuss recent results on the complexity of list homomorphism problems to digraphs. The focus will be on reflexive and irreflexive digraphs whose underlying graphs are trees, cycles, and cliques, as well as on partial orders.

NEW RESULTS ON INTEGRAL GRAPHS

Pavel Híc*, Milan Pokorný

The notion of integral graphs was first introduced by F. Harary and A. J. Schwenk in 1974. A graph G is called integral if all the zeros of the characteristic polynomial $P(G, x)$ are integers. In general, the problem of characterizing of integral graphs seems to be difficult. That is why some special classes of integral graphs are studied.

Integral trees are one important class of these graphs. We constructed the first known integral trees of diameter 7. It is known that an integral tree of diameter 7 cannot be balanced. We shall code a balanced tree of diameter $2k$ by the tree $T(n_k, n_{k-1}, ..., n_1)$, where n_j $(j = 1, 2, ..., k)$ denotes the number of successors of a vertex at the distance $k - j$ from the centre. Let the tree

$$T(n_k, n_{k-1}, ..., n_1)\Theta T(m_j, m_{j-1}, ..., m_1)$$

be obtained by joining the centre w of $T(n_k, n_{k-1}, ..., n_1)$ and the centre v of $T(m_j, m_{j-1}, ..., m_1)$ with a new edge. This tree is denoted by $T(n_1, n_2, ..., n_{k-1}, n_k; 1; m_j, m_{j-1}, ..., m_1)$. We proved that the trees

$T(49, 480, 270; 1; 270, 420, 64)$

$T(25, 264, 504; 1; 504, 220, 36)$

$T(3136, 5328, 1140; 1; 1140, 5700, 3136)$

$T(625, 4704, 1188; 1; 1188, 4900, 576)$

are integral.

Complete n-partite graphs are another interesting class of integral graphs. Only complete bipartite and complete tripartite integral graphs were known so far. We constructed the first known infinite class of complete 4-partite graphs, which is $K_{117q,261q,352q,495q}$, where $q \in N$.

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THE CROSSING NUMBER OF A PROJECTIVE GRAPH IS QUADRATIC IN THE FACE–WIDTH

Isidoro Gitler, Petr Hliněný*,
Jesus Leanos, Gelasio Salazar

We show that for each integer $g \geq 0$ there is a constant $c_g > 0$ such that every graph that embeds in the projective plane with sufficiently large face–width r has crossing number at least $c_g r^2$ in the orientable surface Σ_g of genus g (where Σ_0 is the ordinary plane). As a corollary, we give a polynomial time constant factor approximation algorithm for the crossing number of projective graphs with bounded degree.

Invited lecture

GRAPH THEORY AS AN INTEGRAL PART OF MATHEMATICS

Peter Horák

Nobody doubts the significance of the applications of graph theory to other sciences (e.g. electrical engineering, psychology, economics) or to real life problems. However, in this talk, we concentrate on a different type of application; the possibility of utilization of graph theory methods in other branches of mathematics. To demonstrate this kind of applications, graph theoretical proofs of well-known theorems in areas such as set theory, number theory and analysis will be presented.

DECOMPOSITION OF BIPARTITE GRAPHS INTO CLOSED TRAILS

Sylwia Cichacz, Mirko Horňák*

Let $\mathrm{Lct}(G)$ be the set of all lengths of closed trails that are present in a graph G with all vertices of even degrees. Further, let $\mathrm{Sct}(G)$ be the set of all sequences of terms of $\mathrm{Lct}(G)$ that add up to $|E(G)|$. A sequence $(t_1, \ldots, t_p) \in \mathrm{Sct}(G)$ is G-realisable if there is a sequence $(T_1, \ldots, T_p)$ of pairwise edge-disjoint closed trails in G such that T_i is of length t_i for $i = 1, \ldots, p$. The graph G is arbitrarily decomposable into closed trails provided all sequences in $\mathrm{Sct}(G)$ are G-realisable. It is proved that if $a \geq 1$ is an odd integer and $M_{a,a}$ is a perfect matching in $K_{a,a}$, then the graph $K_{a,a} - M_{a,a}$ is arbitrarily decomposable into closed trails.

COMPLEXITY OF APPROXIMATION FOR 3-EDGE-COLORING OF CUBIC GRAPHS

Katarína Horváthová*, Martin Kochol,
Naďa Krivoňáková, Silvia Smejová

The problem to find a 4-edge-coloring of a 3-regular graph is solvable in polynomial time but an analogous problem for 3-edge-coloring is NP-hard. To make the gap more precise, we study complexity of approximation algorithms for invariants measuring how far is a 3-regular graph from having a 3-edge-coloring. We show that it is an NP-hard problem to approximate such invariants with an error $O(n^{1-\epsilon})$, where n denotes the order of the graph and $0 < \epsilon < 1$ is a constant.

ON (k, l)-RADIUS OF RANDOM GRAPHS

MONIKA HORVÁTHOVÁ

Problems of distance are frequently considered in graph theory. It is useful to know the diameter, the radius and the center of associated graphs. In this contribution we introduce (k, l)-radius which generalizes basic graph invariants connected with distance. We prove that for any fixed pair k, l the (k, l)-radius is equal to $2\binom{k}{2} - \binom{l}{2}$ for almost all graphs. Since for $k = 2$ and $l = 0$ the (k, l)-radius is equal to the diameter, our result is a generalization of the known fact that almost all graphs have diameter two.

A NEW FAMILY OF GRACEFUL TREES

PAVEL HRNČIAR*, GABRIELA MONOSZOVÁ

A graceful labelling of an undirected graph G with n edges is a one-to-one function from the set of vertices of G to the set $\{0, 1, 2, \ldots, n\}$ such that the induced edge labels are all distinct (the label of an edge is the absolute value of the difference between the labels of its end-vertices). The famous Graceful Tree Conjecture (GTC) states that every tree has a graceful labelling. Nevertheless, in spite of a big effort to prove GTC, it is still wide open. There are numerous results proving the conjecture for some very specific classes of trees.

We introduce a new family of trees. Consider a family of stars. Add a new vertex. Join one end-vertex of each star with this new vertex by a path of length h. The tree so obtained will be called a generalized banana tree and for $h = 1$ it is known as a banana tree.

Theorem *Every generalized banana tree is graceful.*

THREE NP-COMPLETE PROBLEMS IN SEIDEL'S SWITCHING

Eva Jelínková

Seidel's switching is a graph operation which makes a given vertex adjacent to precisely those vertices to which it was non-adjacent before, while keeping the rest of the graph unchanged. Two graphs are called switching-equivalent if one can be made isomorphic to the other by a sequence of switches.

In this talk, we show the NP-completeness of the problem Switch-cn-Clique for each $c \in (0, 1)$: determine if a graph G is switching-equivalent to a graph containing a clique of size at least cn, where n is the number of vertices of G. We also prove the NP-completeness of the problems Switch-Max-Edges and Switch-Min-Edges which decide if a given graph is switching-equivalent to a graph having at least or at most a given number of edges, respectively.

ON THE EXISTENCE OF SPECIFIC STARS IN PLANAR GRAPHS

Jochen Harant, Stanislav Jendroľ*

Given a graph G, a $(k; a, b, c)$-star in G is a subgraph isomorphic to a star $K_{1,3}$ with a central vertex of degree k and three leaves of degrees a, b and c in G. The main result of the paper is:

Every planar graph G of minimum degree at least 3 contains a $(k; a, b, c)$-star with $a \leq b \leq c$ and (i) $k = 3$, $a \leq 10$, or (ii) $k = 4$, $a = 4$, $4 \leq b \leq 10$, or (iii) $k = 4$, $a = 5$, $5 \leq b \leq 9$, or (iv) $k = 4$, $6 \leq a \leq 7$, $6 \leq b \leq 8$, or (v) $k = 5$, $4 \leq a \leq 5$, $5 \leq b \leq 6$ and $5 \leq c \leq 7$, or (vi) $k = 5$ and $a = b = c = 6$.

Our result is in some cases best possible and in the remaining ones it differs by at most one from the optimum.

PRACTICAL USE OF FIEDLER VECTOR TO GRAPH PARTITIONING

Pavla Kabelíková

For parallel computing, large computational graphs are partitioned into subdomains and distributed over individual processors. Finding partitioning with minimum edge cut is, in general, an NP–complete problem. We will present suitable heuristics for graph partitioning based on spectral methods. Spectral methods for graph partitioning have been known to be robust but computationally expensive, however, recent development of hardware and software makes these methods interesting for practical applications. The separator produced by spectral methods is related to the Fiedler value (the second smallest eigenvalue of the discrete Laplacian matrix which is associated with the adjacency structure of graph) and corresponding eigenvector – the Fiedler vector.

COVERING GRAPHS BY RAINBOW FORESTS

Tomáš Kaiser*, Matthias Kriesell

Let G be a graph with a (not necessarily proper) coloring of its edges such that the size of each color class is at most m. We call a subgraph of G *rainbow* if it contains at most one edge of each color. We discuss results and problems related to the following question: If the edges of G can be covered by k forests, what is the least number of rainbow forests covering the edge set of G?

The talk is based on joint work with Matthias Kriesell.

ARCHIMEDEAN SOLIDS OF GENUS ONE

Ján Karabáš*, Roman Nedela

By Steinitz theorem, a graph Γ is polyhedral if an only if it is planar, simple and 3-connected. Archimedean solids can be viewed as polyhedral maps sharing a combinatorial symmetry. The classification of Archimedean solids seems to be "not so far" from the classification of regular maps.

A spherical Archimedean solid is a three-dimensional convex polyhedron – a solid which consists of a collection of polygons (faces) such that a local permutation of faces in a vertex v (a local type) does not depend on the choice of v. Thus, the underlying graph of a spherical Archimedean solid is vertex-transitive. Toroidal Archimedean solids are quotients of the uniform (vertex-transitive) tilings of the Euclidean plane described e.g in Grünbaum's monograph. Each tiling gives rise to an infinite family of toroidal Archimedean solids.

In recent time the method which gives rise to a classification of (generalized) Archimedean solids of genus greater than one was developed. We would like to discuss the using of this method in genera zero and one to set the classification in the same terms as is was done in higher genera.

EMBEDDINGS OF TABLES OF GROUPS OF ORDER 8

M.J. Grannell, T.S. Griggs, M. Knor*

We consider orientable triangulations by the complete tripartite graph $K_{n,n,n}$. Such embeddings correspond to biembeddings of Latin squares. Formerly, using a computer, we obtained all such embeddings for $n \leq 7$, while now we concentrated on the case $n = 8$. We focussed our attention to the tables (Latin squares) of the 5 groups of order 8 and we found all their embeddings. Some features of these embeddings are discussed.

ON FACTORIZATIONS OF COMPLETE GRAPHS INTO SPANNING TREES WITH AT MOST FOUR NONLEAVE VERTICES

Dalibor Frонček, Petr Kovář*,
Michael Kubesa

The spanning trees factorization of complete graphs is a well established topic which is far from being closed. Only few classes of trees are classified whether they factorize complete graphs.

In this talk we give several necessary conditions for a spanning tree to factorize a complete graph on $2n$ vertices. Then we present the complete characterization of spanning trees with at most four nonleave vertices that factorize K_{2n}.

TELLEDOMINATION IN CHORDAL GRAPHS

Jan Kratochvíl

REDUCTION OF MATRICES DETERMINING RESTRICTIONS FOR COUNTEREXAMPLES TO 5-FLOW CONJECTURE

Martin Kochol, Katrína Horváthová,
Naďa Krivoňáková*, Silvia Smejová

Recently was developed a method which determines graphs that cannot be subgraphs of a smallest counterexample to the 5-flow conjecture. This method is based on comparing ranks of two matrices of large size.

In this paper we improve the previous methods from Kochol and present an approach how to reduce the size of the matrix used in the process. In particular, in order to show that the smallest counterexample to the 5-flow conjecture has no circuit of length 7, we need to deal with matrix of size 819×162 in [M.Kochol: Restrictions on smallest counterexamples to the 5-flow conecture]. In this paper we reduce the size of the matrix into 317×110 to get the same result.

SURVEY OF METHODS FOR FACTORIZATIONS OF COMPLETE GRAPHS INTO SPANNING TREES

Dalibor Fronček, Petr Kovář,
Tereza Kovářová, Michael Kubesa*

Until recently it was only known that some symmetric trees factorize a complete graph K_{2n}. A symmetric tree is a tree T with an edge $e = xy$ such that there exists an automorphism φ of T so that $\varphi(x) = y$ and $\varphi(y) = x$. Therefore, we began to examine factorizations of complete graphs into non-symmetric spanning trees. We completely characterized spanning trees with at most four non-leave vertices that factorize K_{2n}. We want to present here all methods, which we are using.

THE SPECTRAL PROPERTIES OF THE CARTESIAN PRODUCT

Clemens Brand, Wilfried Imrich,
Tomáš Kupka*

Professor Brand suggested to explore whether the spectrum of the Laplacian of a graph can be used to find its factorization with respect to the Cartesian product. Our approach uses Fiedler's vector (the eigenvector that correponds to the second lowest eigenvalue) of the Laplacian to distinguish the layers of factors. We also show how to use the information provided by the spectrum for the recognition of approximate Cartesian products.

A CLOSURE CONCEPT IN $K_{1,r}$-FREE GRAPHS

Roman Kužel

A graph G is called *$K_{1,r}$-free* if G has no $K_{1,r}$ as an induced subgraph. A hypergraph H is called *k-uniform* if all its edges have cardinality k. Clearly a 2-uniform hypergraph is a graph in a usual sense. A *line graph* of a hypergraph $H = (V, E)$ is a graph $L(H)$ with vertex set E and with two vertices adjacent in $L(H)$ iff the corresponding edges of H have a nonempty intersection. Let v be a vertex of a graph G, we say that v is *locally connected* if the neighborhood of v in G induces a connected subgraph in G. If a locally connected vertex v induces a non complete subgraph we call the vertex v *eligible*. Let G be a graph and $v \in V(G)$ be its eligible vertex. A graph G_v obtained from a graph G by adding all missing edges into a neighborhood of v is called a *local completion* of G at vertex v. A *closure* of a graph G is a graph $\mathrm{Cl}(G)$ obtained by repeating of an operation of a local completion at eligible vertices until no other eligible vertex remains. A *k-walk* in a graph G is a closed walk visiting each vertex at most k times, where $k \geq 1$ is an integer. Note, that in our notations a k-walk in a graph G need not to span all vertices of G. We can then view a cycle in a graph as an 1-walk. A *k-circumference* of a graph G is a cardinality of a vertex set of a k-walk $W \subset G$ spanning maximum possible number of vertices of G. We denote by $c_k(G)$ the k-circumference of a graph G. We show the following generalization of the well-known Ryjáček's closure concept in $K_{1,3}$-free graphs.

Theorem *Let $r \geq 3$ be an integer and let G be a $K_{1,r}$-free graph. Then*

(i) $c_{r-2}(G) = c_{r-2}(\mathrm{Cl}(G))$,

(ii) Cl(G) *is unique,*

(iii) there is an $(r-1)$-uniform hypergraph H such that $\mathrm{Cl}(G) = L(H)$.

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Invited lecture

GRAPH COLOURINGS IN STEGANOGRAPHY

Petr Lisoněk

Steganography is the science of information hiding. The sender embeds a secret message into the cover object (e.g., a multimedia file) by slightly distorting it in a way that enables the intended recipient to retrieve the hidden message; at the same time the very existence of the hidden message should be impossible to detect by any third party.

The main goal of Steganography is to design schemes with high embedding efficiency, which is the ratio between the amount of the communicated information and the amount of introduced distortion.

We will show that one class of steganography schemes can be described by graphs that admit a so-called "rainbow colouring." A rainbow colouring of a k-regular graph G is a proper vertex colouring of G with $k+1$ colours such that, for each $v \in V(G)$, all neighbours of v receive distinct colours. The Steganography application requires rainbow colouring functions that have a very low complexity, say $O((\log |V(G)|)^c)$ space and time.

As an explicit example we will note that the d-dimensional integer lattice graph $\mathbb{Z}^d$ admits a rainbow colouring for each d. In a combination with linear codes over finite fields this specific rainbow colouring can be extended to steganographic schemes that have a better embedding efficiency than the previously used schemes.

REAL FLOW NUMBER AND THE CYCLE RANK OF A GRAPH

Robert Lukoťka*, Martin Škoviera

We establish a relationship between the real (circular) flow number of a graph and its cycle rank. We show that a connected graph with real flow number $p/q + 1$ where p and q are two relatively prime numbers must have cycle rank at least $p + q - 1$. A special case of this result yields that the real flow number of a 2-connected cubic graph with chromatic index 4 and order at most $8k + 4$ is bounded from below by $4 + 1/k$. Using this bound we prove that the real flow number of the Isaacs snark I_{2k+1} equals $4 + 1/k$, completing the upper bound due to Steffen (J. Graph Theory **36** (2001), 24–34).

DECOMPOSITION OF REGULAR t-BALANCED CAYLEY MAPS FROM CYCLIC GROUPS

Martin Mačaj

We introduce notion of product of Cayley maps and show some basic properties of such products. As an application we show that any regular t-balanced Cayley map from a cyclic group can be expressed as a product of regular balanced and regular anti-balanced Cayley map.

HYPOHAMILTONIAN CUBIC GRAPHS OF GIRTH 7

Edita Máčajová*, Martin Škoviera

To decide whether a given graph is hamiltonian is a well known NP-complete problem and it is NP-complete even within the family of cubic graphs. *Hypohamiltonian graphs* – graphs for which the removal of an arbitrary vertex leads to a hamiltonian graph – lie on the border between the class hamiltonian graphs and its complement. Better understanding of hypohamiltonian graphs can therefore lead to demarcation line for hamiltonian graphs.

Roughly speaking, the higher girth and cyclic connectivity, the harder to find a hypohamiltonian graph with these parameters. Moreover, Thomassen conjectured, that there exists a constant k (possibly $k = 8$) such that every cyclically k-edge-connected cubic graph is hamiltonian. Interesting in this sense is the Coxeter graph, having both girth and cyclic connectivity 7, for it is the only highly connected graph of girth at least 7 that is known to be hypohamiltonian.

We provide a construction, which uses as building blocks a graph derived from the Coxeter graph, giving rise to an infinite family of cubic hypohamiltonian graphs of girth 7, with cyclic connectivity 4, 5, or 6.

Invited lecture

GRAPHS OF ORDER CLOSE TO MOORE BOUND: OVERVIEW AND OPEN PROBLEMS

Mirka Miller

The degree/diameter problem is to determine, for each d and k, the largest order $n_{d,k}$ of a graph (respectively, digraph) of maximum degree d (respectively, maximum out-degree d) and diameter at most k. It is easy to see that $n_{d,k} \leq M_{d,k}$, where $M_{d,k}$ is the Moore bound given by

$$M_{d,k} = 1 + d + d(d-1) + \cdots + d(d-1)^{k-1}$$

for undirected graphs, and

$$M_{d,k} = 1 + d + d^2 + \cdots + d^k$$

for directed graphs.

Graphs and digraphs of order equal to the Moore bound are called Moore graphs (respectively, Moore digraphs). Since there are very few Moore graphs and digraphs, it becomes interesting to study (d,k,δ)-graphs (respectively, (d,k,δ)-digraphs), that is, graphs (respectively, digraphs) of order δ less than the Moore bound.

In this talk we give an overview of the problem and present several interesting open problems in this area.

ENUMERATION OF MAPS REGARDLESS GENUS

Alexander Mednykh and Roman Nedela*

A combinatorial map is an abstraction of geometric map - graph cellularly embedded into a surface. Map enumeration is related with classical results of Hurwitz (1981) on the number of non-isomorphic covers over the sphere having simple branch points of order two and of Hall (1949) who derived a formula for the number of subgroups of given index in a free group. In 1961-1964 Tutte developed methods for counting various families of planar maps. Tutte's pioneering works had a lot of successors. In our talk we apply a classical combinatorial approach to enumerate rooted maps with given number of edges regardless genus. Next we apply a new technique developed by the authors to compute isomorphism classes of such maps. The results gives information on the ratio between the number of reflexible and chiral maps with given number of edges. No result of this sort was known.

ON DOMINATING CLIQUES IN RANDOM GRAPHS

Michal Demetrian, Martin Nehéz*,
Daniel Olejár

The phase transition phenomenon, originally observed as a physical effect, used to be often emerged also in discrete structures like random graphs. The most frequent property of random graphs which have been studied with relation to the phase transitions is the connectivity. Our paper deals with another interesting graph problem that is an emerging of dominating cliques in a standard Erdös-Rényi random graph model.

Given a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a *dominating set* of G if each node $v \in V$ is either in S or is adjacent to a node in S. A *clique* in G is a maximal set of mutually adjacent nodes of G, i.e., it is a maximal complete subgraph of G. If a subgraph S induced by a dominating set is a clique in G then S is called a *dominating clique* in G. Given a fixed constant p, $0 < p < 1$, let us consider the standard Erdös-Rényi random graph model denoted by $\mathbb{G}(n,p)$.

Theorem *Let $\mathbb{L}x$ denote $\log_{1/(1-p)} x$. Let r be an order of the clique such that $\lfloor \log_{1/p} n \rfloor \le r \le \lceil 2\log_{1/p} n \rceil$. Let $\delta(n) : \mathbb{N} \to \mathbb{N}$ be an arbitrary slowly increasing function such that $\delta(n) = o(\log n)$ and let $G \in \mathbb{G}(n,p)$ be a random graph. Then it holds:*

1. *If $p > 1/2$, then an r-node clique is dominating in G almost surely;*
2. *If $p \le (3-\sqrt{5})/2$, then an r-node clique is not dominating in G almost surely;*
3. *If $(3-\sqrt{5})/2 < p \le 1/2$, then an r-node clique:*
    - *is dominating in G almost surely, if $r \ge \mathbb{L}n + \delta(n)$,*
    - *is not dominating in G almost surely, if $r \le \mathbb{L}n - \delta(n)$,*
    - *is dominating with a finite probability $f(p)$ for a suitable function $f : [0,1] \to [0,1]$, if $r = \mathbb{L}n + O(1)$.*

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ON THE RECOGNITION OF PC-GRAPHS WITH HIGH GIRTH

JAN KRATOCHVÍL, MARTIN PERGEL*

Intersection graphs are graphs having representations by set-systems of certain types. Each set corresponds to a vertex and two vertices are adjacent whenever the corresponding sets have nonempty intersection. As for each graph can be generated some intersection representation [3], we are interested only in some particular classes. The recognition problem (to decide whether a given graph has an appropriate intersection representation) is a very important problem, as several efficient algorithms solving generally hard problems require this representation.

Polygon-circle (or PC) graphs are intersection graphs of polygons inscribed into a circle in a plane [4]. PC-graphs generalize many other intersection-defined classes (e.g., interval graphs, circle graphs, circular-arc graphs) whose recognition problem is known to be polynomially solvable. As it is known a polynomial algorithm [1] finding maximum weight clique and independent set even for graphs of interval filaments (generalization of PC-graphs), while in [2] it is proved that PC-graphs are near-perfect, it is important to ask, how efficiently can this class be recognized.

We introduce a polynomial-time algorithm for the recognition of polygon-circle graphs with girth at least 5, as well we establish polynomial reduction showing that the general recognition problem (for PC-graphs) is NP-complete.

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EQUITABLE SPECIALIZED BLOCK-COLOURINGS OF STEINER TRIPLE SYSTEMS

Mario Gionfriddo, Peter Horak,
Lorenzo Milazzo, Alexander Rosa*

In specialized block-colourings of Steiner triple systems, the triples incident with any element are coloured according to a given partition of the replication number. Such colourings are equitable if the partition is equitable, that is, the difference between any two of its parts is at most one. We examine equitable colourings for partitions having two or three parts, and completely characterize the former.

(This is a joint work with Mario Gionfriddo, Peter Horak and Lorenzo Milazzo.)

GRAPHS WITH ODD CYCLES OF LENGTH 5 AND 7 ARE 3-COLOURABLE. GENERALIZATIONS POSSIBLE?

TOMÁŠ KAISER, ONDŘEJ RUCKÝ*,
RISTE ŠKREKOVSKI

Let $L(G)$ be the set of odd cycle lengths of a graph G. A well known conjecture of Bollobás and Erdős said that $|L(G)| = k$ implied $\chi(G) \leq 2k + 2$; it was later strengthened by Gallai and finally proved by Gyárfás to be true in the following form.

Theorem *If G is a 2-connected graph with minimum degree at least $2k + 1$ then $|L(G)| = k \geq 1$ implies $G = K_{2k+2}$.*

We are interested in a particular case when $|L(G)| = 2$ and $3 \notin L(G)$. The theorem then asserts the chromatic number of G is at most 5. But it seems the chromatic number is actually significantly lower than this estimation, namely we believe it is always equal to 3.

We are able to prove this conjecture for a very special case when $L(G) = \{5, 7\}$ so far. Our proof consists of two main stages. The first one is proved for a more general circumstance when $|L(G)| = 2$ and the two odd cycle lengths are two consecutive odd numbers, and it seems to be only a matter of more case analysis to extend it to the desired generality. Unfortunately, the next step of the proof is closely tailored to the considered particular setting; although it could be apparently proved for several other special cases like $L(G) = \{7, 9\}$ for example, we are pretty sure extending of it to the fully general case needs a quite new approach.

FORBIDDEN SUBGRAPHS THAT IMPLY 2-FACTORS

J.R. Faudree, R.J. Faudree, Zdeněk Ryjáček*

The connected forbidden subgraphs and pairs of connected forbidden subgraphs that imply a 2-connected graph is hamiltonian have been characterized by Bedrossian, and extensions for general graphs of order at least 10 were proved by Faudree and Gould. We give a complete characterization of connected forbidden subgraphs and pairs of connected forbidden subgraphs that imply a 2-connected graph of order at least 10 has a 2-factor. In particular it will be shown that the characterization for 2-factors is very similar to that for hamiltonian cycles, except there are seven additional pairs. In the case of graphs of all possible orders, there are four additional forbidden pairs not in the hamiltonian characterization, but the claw is a part of each pair.

INDUCED HEREDITARY GRAPH CLASSES AND CLOSURE OPERATION

JAN BROUSEK, KHIKMAT SABUROV*

Let P_i be the path with i vertices, C be the claw $K_{1,3}$ and N the net, i.e. the only connected graph with degree sequence 333111. It is known that if a graph G is 2-connected and CP_6-free, then its closure cl(G) is CN-free. Let N_{112} be the eiffel, i.e. the graph obtained by subdividing one pendant edge of the net with a vertex of degree 2. In the talk we show that if G is a 2-connected CP_7-free graph, then cl(G) is CN_{112}-free or cl(G) belong to some 4 fully described infinite classes of exceptions. We also show that there is a similar relation for 2-connected CN_{112}-free and CN-free graphs (with $\mathcal{C}_1^N, \mathcal{C}_2^N, \mathcal{C}_3^N, \mathcal{C}_4^N, \mathcal{C}_{3a}^N$ and $\mathcal{C}_{4a}^N$ at exceptions). From these propositions it follows that if G is 3-connected CP_7-free graph, then cl(G) is CN-free.

SUPERARITHMETIC GRAPHS

ANDREA SEMANIČOVÁ

A graph is called d-arithmetic (d-superarithmetic) if it admits a labelling of the edges by pairwise different (and consecutive) integers such that the vertex sums form an arithmetic sequence with the difference d. In this talk we will present some necessary conditions for d-arithmetic and d-superarithmetic graphs. We will also describe some constructions of d-arithmetic and d-superarithmetic graphs.

Invited lecture

HEREDITARY PROPERTIES OF GRAPHS

Gabriel Semanišin

We shall provide a survey of results concerning hereditary properties of graphs. The presented results were obtained mainly in co-operation with P. Mihók, R. Vasky, M. Frick, M. Borowiecki, I. Broere, A . Farrugia, R.B. Richter.

The original graph colouring problem can be reformulated in the following way: Determine the minimum number of independent sets to which the vertex set of G can be partitioned. A natural generalization of this problem can be obtained in such a way that we replace the adjacency constraint by some constraint on the structure of monochromatic subgraphs. A convenient language that may be used for formulating problems of graph colouring in a general setting is the language of reducible monotone properties.

In our contribution we shall discuss:

- the lattice of hereditary properties and its utilisation for the comprehensive study of generalised colouring;
- a characterisation of hereditary properties of graphs in terms of forbidden subgraphs, maximal graphs and generators;
- an extremal problems related to hereditary properties of graphs;
- graph invariants and their relationship to the chains of hereditary properties;
- minimal reducible bounds and their utilisation for an evaluation and comparison of different colouring and partitioning results;
- the problem of unique factorization of hereditary properties of graphs and its various modifications;
- complexity results concerning hereditary properties of graphs;
- an application of formal concept analysis to the field of hereditary properties of graphs.

CONSTRUCTIONS OF UNIVERSAL PARTS OF SOME COMPLETE GRAPHS

Anna Kędzior, Zdzisław Skupień*

Main result: Universal tth parts of K_n exist for each n and $t \leq 6$. Being universal means that respective t-packings of a part into K_n can leave all possible edge remainders R such that the size $|R|$ is the smallest possible. Due to recursion, the proof reduces to presenting universal parts for a few orders n. In particular, a fourth part of K_{11} and a sixth part of K_{15}, both universal, will be constructed. Then $|R| = 3$. Therefore the proof of either construction requires five packings only (since three edges can induce five nonisomorphic graphs only).

ON SOME NEW TYPES AND KINDS OF POLYHEDRA NON-INSCRIBABILITY

Sergej Ševec

Types of polyhedra are studied that are non-inscribable in the spherical shell. A general sufficient condition is formulated. Employing it, a large class of polyhedra is proved to be spherical-shell non-inscribable in addition to those non-inscribable in the sphere. Several illustrations of applying the sufficient condition proved (as well as the proof techniques used) are presented on the examples of spherical-shell non-inscribability calculation for particular polyhedra types. In the second part of the paper, the relationship between (non-) inscribability in the spherical shell and (non-) inscribability in the sphere is focused and traced more closely. In consequence, a new kind of (non-) inscribability is defined, that interpolates these two ones.

ON THE EDGE-RAINBOWNESS OF A FAMILY OF PLANE GRAPHS

Erika Škrabuľáková

A face of an edge colored plane graph is called *rainbow* if all its edges receive distinct colors. The maximum number of colors used in edge coloring of a connected plane graph G with no rainbow face is called *the edge - rainbowness* of G. The graph H is said to be a *face factor* of a plane graph G, if $V(H) = V(G)$, $E(H) \subseteq E(G)$ and for every face $f \in F(G)$ there is a face $f' \in F(H)$, such that f is involved in f'. A face factor H of a connected plane graph G is called a *bridge face factor* if every face of H is incident with a bridge. It is proved [1] that the edge rainbowness of G equals to the number of edges of a maximum connected bridge face factor H of G.

Shin-Shin Kao and Lih-Hsing Hsu defined spider web network $SW(m,n)$ as follows:
For even integers $m \geq 4$, $n \geq 2$, $SW(m,n)$ is a graph with the vertex set $\{(i,j); 0 \leq i < m, 0 \leq j < n\}$ such that (i,j) and (k,l) are adjacent if they satisfy one of the following conditions:

1. $i = k$ and $j = l + 1$ or $i = k$ and $j = l - 1$; or
2. $j = l$ and $k = [i+1]_m$ if $i + j$ is odd or $j = n - 1$; or
3. $j = l$ and $k = [i-1]_m$ if $i + j$ is even or $j = 0$.

In the talk we show upper and lower bounds on the edge rainbowness of spider graphs $SW(m,n)$ for every possible m and n.

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TOUGHNESS TRESHOLD FOR THE EXISTENCE OF 2-WALKS IN K_4-MINOR FREE GRAPHS

ZDENĚK DVOŘÁK, DANIEL KRÁĽ[1], JAKUB TESKA*

A 2-walk is a closed spanning trail which uses every vertex at most twice. The *toughness* of a non-complete graph is $t(G) = min(\frac{|S|}{c(G-S)})$, where the minimum is taken over all nonempty vertex sets S, for which $c(G-S) \geq 2$ and $c(G-S)$ denotes the number of components of the graph $G-S$. We show that every K_4-minor free graph with toughness $t(G) > \frac{4}{7}$ has a 2-walk. We also give an example of a $\frac{4}{7}$-tough K_4-minor free graph with no 2-walk.

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SQUARE OF METRICALLY REGULAR GRAPHS

VLADIMÍR VETCHÝ

Let X be a finite set, $n := |X| \geq 2$. For an arbitrary natural number D let $\mathbf{R}= \{R_0, R_1, \dots, R_D\}$ be a system of binary relations on X. A pair $(X,\mathbf{R})$ will be called *an association scheme* with n classes if and only if it satisfies the axioms $A1 - A4$:

A1. The system $\mathbf{R}$ forms a partition of the set X^2 and R_0 is the diagonal relation, i.e. $R_0 = \{(x,x); x \in X\}$.

A2. For each $i \in \{0,1,\dots,D\}$ it holds $R_i^{-1} \in \mathbf{R}$.

A3. For each $i,j,k \in \{0,1,\dots,D\}$ it holds
$(x,y) \in R_k \wedge (x_1,y_1) \in R_k \Rightarrow p_{ij}(x,y) = p_{ij}(x_1,y_1)$, where $p_{ij}(x,y) = |\{z; (x,z) \in R_i \wedge (z,y) \in R_j\}|$.
Then define $p_{ij}^k := p_{ij}(x,y)$ where $(x,y) \in R_k$.

A4. For each $i,j,k \in \{0,1,\dots,D\}$ it holds $p_{ij}^k = p_{ji}^k$.

The set X will be called the *carrier* of the association scheme $(X,\mathbf{R})$. Especially, $p_{i0}^k = \delta_{ik}$, $p_{ij}^0 = v_i\delta_{ij}$, where δ_{ij} is the Kronecker-Symbol and $v_i := p_{ii}^0$, and define $P_j := (p_{ij}^k)$, $0 \leq i,j,k \leq D$.

Given an undirected graph $G = (X,E)$ of diameter D we may now define $R_k = \{(x,y); d(x,y) = k\}$, where $d(x,y)$ is the distance from the vertex x to the vertex y in the standard graph metric. If $(X,\mathbf{R})$ gives rise to an association scheme, the graph G is called *metrically regular* (sometimes also called *distance regular*) and p_{ij}^k are said to be its *parameters*. In particular, a metrically regular graph with diameter $D = 2$ is called *strongly regular*.

Let $G = (X,E)$ be an undirected graph without loops and multiple edges. *The second power* (or *square* of G) is the graph $G^2 = (X,E')$ with the same vertex set X and in which mutually different vertices are adjacent if and only if there is at least one path of the length 1 or 2 in G between them.

The necessary conditions for G to have the square G^2 metrically regular are found and some constructions of those graphs are solved.

CHOOSABILITY OF COMPLETE MULTIPARTITE GRAPHS

Tomáš Vetrík

A list assignment to the vertices of a graph G is the assignment of a list $L(v)$ of colors C to every vertex v of G. A k-list assignment is a list assignment such that $|L(v)| \geq k$ for every vertex v. A proper L-coloring of G is a function $f : V(G) \to C$ such that $f(v) \in L(v)$ for all $v \in V(G)$ and $f(u) \neq f(v)$ for every $uv \in E(G)$. If for every k-list assignment L, there exists an L-coloring, then G is k-choosable. The choice number $ch(G)$ is the smallest number k such that G is k-choosable.

We focus on choosability of complete multipartite graphs. Let $K_{n_1,n_2,\ldots,n_r}$ be the complete multipartite graph with r partite sets of order $n_1, n_2, \ldots, n_r$. Our main result shows that a complete r-partite graph G consisting of one partite set of order $(t+2)(t+3)/2$ and $r-1$ partite sets of order two is $(r+t)$-choosable. We also present results on upper and lower bounds for choice number of complete multipartite graphs with partite sets of equal sizes.

Invited lecture

ON 1, 2 CONJECTURE

Jakub Przybyło, Mariusz Woźniak*

A *k-total-weighting* of a simple graph G is an assignment of an integer weight, $w(e), w(v) \in \{1, \ldots, k\}$ to each edge e and each vertex v of G. The k-total-weighting is *neighbour-distinguishing* if for every edge uv, $w(u) + \sum_{e \ni u} w(e) \neq w(v) + \sum_{e \ni v} w(e)$. If it exists, we say that G *permits* a neighbour-distinguishing k-total-weighting. The smallest k for which G permits a neighbour-distinguishing k-total-weighting we denote by $\tau(G)$.

Similar parameter, but in the case of an *edge* (not total) weighting was introduced and studied in by Karoński, Łuczak and Thomason. They asked if each, except for a single edge, simple connected graph permits a *neighbour-distinguishing* 3-*edge-weighting*, and showed that this statement holds e.g. for the 3-colourable graphs. It is also known that each *nice* (not containing a connected component which has only one edge) graph permits a neighbour-distinguishing 16-edge-weighting, hence this parameter is finite.

Note that if a graph permits a neighbour-distinguishing k-edge-weighting, then it also permits a neighbour-distinguishing k-total-weighting (it is enough to put ones at all vertices), hence we obtain an upper bound $\tau(G) \leqslant 16$ for all graphs and $\tau(G) \leqslant 3$ for all graphs if the conjecture by Karoński, Łuczak and Thomason holds.

Therefore, we ask if, maybe, the weights 1 and 2 are enough in the case of a total-weighting.

It is also worth mentioning here that our reasonings correspond with the recent study of Bača, Jendrol, Miller and Ryan. They introduced and studied a parameter called a *total vertex irregularity strength*, which is the smallest k for which there exists a k-total-weighting such that each vertex of a graph receive a different colour, i.e. $w(u) + \sum_{e \ni u} w(e) \neq w(v) + \sum_{e \ni v} w(e)$ for each (not only neighbouring ones) pair of vertices u, v.

This parameter, as well as the other mentioned in this section, may be viewed as descendants of the well known *irregularity strength* of a graph.

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ON THE LOCAL STRUCTURE OF PLANE GRAPHS EXTREMAL ACCORDING TO WEIGHT OF TRIANGLES

Igor Fabrici, Tomáš Madaras,
Jana Zlámalová*

The weight of a subgraph H of a graph G is defined to be the sum of degrees (with respect to G) of vertices of H. Borodin proved that every plane graph of minimum degree 5 contains a triangular face of weight at most 17, the bound being precise. According to this result, we explore the family of extremal plane triangulations whose faces are of weight at least 17; we show that graphs of this family contain a variety of light (that is, weight-bounded) subgraphs, among them a 5-star with all vertices of degree at most 7, a 6-star with all vertices of degree at most 9 (both these bounds are precise), and a light 11-cycle.