



Invited lecture

**TOWARDS FULLY MULTIVARIATE ALGORITHMICS: SOME NEW
RESULTS AND DIRECTIONS IN PARAMETER ECOLOGY**

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Parameterized complexity is a natural two-dimensional sequel to the familiar one-dimensional framework of P vs NP. The classical framework is one-dimensional, because all that is considered is the single measurement: n , the total input size. The central concept is *polynomial time* (P). Parameterized complexity starts from the premise that there is some relevant secondary measurement: k , that captures other aspects of the challenge that affect computational complexity and opportunities for effective algorithm design. The parameter can be many different things. The central concept in parameterized complexity is the generalization of P termed *fixed-parameter tractability* (FPT), solvability in time $f(k)n^c$, where f is an arbitrary function, and c is a constant. P was initially somewhat controversial — the issue was the generous license given to the exponent of the polynomial. FPT has also been controversial, from the license given to f . However, the natural landscape of computing has been well-behaved with respect to both frameworks, and both are successful. One can reasonably ask: “If parameterized complexity is the two-dimensional sequel, then what is the three-dimensional sequel?” So far, there is no answer. What is required, for a fully multivariate complexity theory, is that dimension 1 is centered on P, that dimension 2 is centered on FPT, that the entire theory is guided by natural concrete problems of practical significance and that the theory proposes questions that are routinely mathematically solvable with the toolkit that the theory helps to develop.

“Parameter ecology” is an awkward name given to a program of exploration of how two secondary measurements interact with overall input size in determining computational complexity. Such exploration is natural, well-motivated, seems to be routinely doable, and to lead to new concrete results, toolkit development and open problems. This program may provide insights into how to build a fully multivariate theory of computational complexity and algorithmics.

The talk will offer: the context of the quest for fully multivariate algorithmics, the parameter ecology program and how this connects to that quest, practical motivations, some sketches of recent results, and perhaps guiding stellar open problems.
