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YOUTH AND SPORTS

Workshop on Applied Mathematics

June 24 2022

EA553, FEECS, VŠB-TUO

Program

Morning Section: Graph colorings
Chairman: Petr Kovář

10^{00} – 11^{55} **Tomáš Madaras** – *Homogeneous graph colorings*

11^{00} – 11^{55} **Igor Fabrici** - *Unique-maximum Colorings of Plane Graphs*

Noon Section: Graph decompositions
Chairman: Petr Kovář

12^{15} - 13^{00} **Dalibor Frončec** – *Decomposition of Complete Graphs into Unicyclic Graphs with Seven Edges*

Abstracts and About speakers

Tomáš Madaras – Homogeneous graph colorings
Faculty of Science, Pavol Jozef Šafárik University in Košice, Slovakia

Abstract: We introduce new kinds of graph colorings which can be described, using a unified framework, in the following way: Given a graph *G* and a list $L = \{ H_1, ..., H_t\}$ of (particular) subgraphs of *G*, a coloring *c* (vertex, edge, total etc.) of *G* is called *homogeneous with respect to L* if each subgraph from *L* sees the same number of colors. We present selected results on homogeneous colorings that are connected with subgraph lists consisting of open vertex neighborhoods, or vertex sets of faces (assumed that underlying graph is embedded to a surface), or else the maximal stars/double stars (for the edge coloring version). The relations to other coloring concepts (such as role colorings or M_q-colorings) are discussed as well.

Igor Fabrici - *Unique-maximum Colorings of Plane Graphs*
Faculty of Science, Pavol Jozef Šafárik University in Košice, Slovakia
(this is joint work with Simona Rindošová)

Abstract: A proper vertex coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices is used exactly once. Wendland (2016) proved that every plane graph is UM 5-vertex-colorable and Lidický, Messerschmidt, and Škrekovski (2018) constructed a plane graph with corresponding chromatic number 5.

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In this talk, we consider following two modifications of the previous coloring:

1. a proper vertex-face coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices and on the face itself is used exactly once,
2. a proper vertex coloring of a plane graph is unique-double-maximum (U2M) if, for every face, the highest color and the second highest color on its vertices are used exactly once,

we compare these two colorings, and prove that every plane graph is UM 8-vertex-face-colorable, UM 10-vertex-face-choosable, and U2M 9-vertex-colorable.

Dalibor Fronček – *Decomposition of complete graphs into unicyclic graphs with seven edges*
Swenson College of Science and Engineering, University of Minnesota Dulth, USA

(this is joint work with Michael Kubesa)

Abstract: An H-decomposition of a graph G is a collection of subgraphs $H_1, H_2, \ldots, H_s$ of G, all isomorphic to H, such that every edge of G belongs to exactly one copy H_i. A unicyclic graph is a graph containing precisely one cycle.
The problem of decomposition of complete graphs has been completely solved for graphs with at most six edges, and with a few exceptions also for graphs with eight edges. However, for graphs with seven edges it was almost untouched except for trees. We therefore take the next natural step and examine decompositions into unicyclic graphs with seven edges.

The obvious necessary condition for such a decomposition into graphs with seven edges is that $n \equiv 0,1 \pmod 7$. We show that the condition is also sufficient for any connected unicyclic bipartite graph H with seven edges when $n \equiv 0 \pmod 7$ and $n \equiv 1 \pmod{14}$ except for several cases when $n=7$ or 8. We may also show some partial results for the non-bipartite case.

For disconnected graphs, we present just partial results for the bipartite case, as the work is still in progress.