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# Online Workshop on Applied Mathematics

November 4 - 5 2021

## ZOOM:

<https://cesnet.zoom.us/j/91519768684?pwd=T3U0dW5NMkR5bIZzRGMwOVJmUVFvQT09>

## Program

### Thursday November 4th 2021

#### **Morning Section: Applied Dynamical Systems**

**Chairman:** Marek Lampart

9<sup>00</sup> - 9<sup>45</sup> **Maciej Capinski** - *Arnold Diffusion in the Three Body Problem*

10<sup>00</sup> - 10<sup>45</sup> **Giuseppe Orlando** - *Nonlinear Economic Cycles: History, Theory and Empirics: Advances on modelling and econometrics for alternative directions in macroeconomics and cycle theories*

11<sup>00</sup> - 11<sup>45</sup> **Nikolay V. Kuznetsov** - *The Theory of Hidden Oscillations and Stability of Dynamical Systems*

#### **Afternoon Section: Reliability and Maintenance**

**Chairman:** Radim Briš

13<sup>30</sup> - 14<sup>30</sup> **Piero Baraldi** - *Predictive Maintenance in Industry 4.0*

14<sup>45</sup> - 15<sup>45</sup> **Eric Chatelet** - *Maintenance Optimization of Wind Farms*

### Friday November 5th 2021

#### **Morning Section: Mathematical Modelling and Numerical Simulations**

**Chairman:** David Horák

9<sup>00</sup> - 10<sup>45</sup> **Ulrich Rüde** - *Scalable Multiphysics Simulations*

#### **Afternoon Section: Graph Colorings and Coverability of Graphs**

**Chairman:** Petr Kovář

14<sup>00</sup> - 14<sup>50</sup> **Riste Škrekovski** - *Some Results on Unique-maximum Coloring of Plane Graphs*

15<sup>00</sup> - 15<sup>50</sup> **Borut Lužar** - *Edge-coloring Subcubic Graphs with Five Colors*

16<sup>00</sup> - 16<sup>20</sup> **Kenny Štorgel** - *Further Extensions of the Grötzsch Theorem*

16<sup>25</sup> - 17<sup>05</sup> **Mirko Petruševski** - *Coverability by Parity Regular Subgraphs*



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## **Abstracts and About speakers**

### **Applied Dynamical Systems**

**Maciej Capinski** - *Arnold Diffusion in the Three Body Problem*

Faculty of Applied Mathematics, AGH University of Science and Technology Krakow, Poland

**Abstract:** In this talk we will give an introduction to the problem of Arnold diffusion, and show that a proof of such phenomenon can be obtained from topological and geometric arguments. We will also present a (rigorous, interval arithmetic based) computer assisted proof of Arnold diffusion in the restricted three body problem.

**Giuseppe Orlando** - *Nonlinear Economic Cycles: History, Theory and Empirics: Advances on modelling and econometrics for alternative directions in macroeconomics and cycle theories*

Department of Economics and Finance, University of Bari Aldo Moro, Italy

**Abstract:** The talk is about the topics discussed in this book *Nonlinearities in Economics*, Springer, Cham, ISBN 978-3-030-70981-5, 2021. The approach is interdisciplinary and argues that the economy has an underlying non-linear structure and that business cycles are endogenous, which allows a greater explanatory power concerning the traditional assumption that dynamics are stochastic and shocks are exogenous.

The first part of this talk will provide a definition of business cycles and present a historical overview of the related models. Then, the focus will be on three popular deterministic models of business cycles: Goodwin, Kalecki, and Kaldor. Finally, a stochastic model called “dynamic stochastic general equilibrium” (DSGE) will be presented.

**Nikolay V. Kuznetsov** - *The Theory of Hidden Oscillations and Stability of Dynamical Systems*

Saint-Petersburg State University, Russia; Institute for Problems in Mechanical Engineering of the Russian Academy of Science, Russia; University of Jyväskylä, Finland

**Abstract:** The development of the theory of global stability, the theory of bifurcations, the theory of chaos, and new computing technologies made it possible to take a fresh look at a number of well-known theoretical and practical problems in the analysis of multidimensional dynamical systems and led to the emergence of the theory of hidden oscillations which represents the genesis of the modern era of Andronov’s theory of oscillations. The theory of hidden oscillations is based on a new classification of attractors as self-excited or hidden. While trivial attractors (equilibrium points) can be easily found analytically or numerically, the search of periodic or chaotic attractors may turn out to be a challenging problem (see, e.g. famous 16<sup>th</sup> Hilbert problem on the number and disposition of limit periodic oscillations in two-dimensional polynomial systems which is still unsolved). Self-excited attractors can be easily discovered when observing numerically the dynamics with initial data from the vicinity of the equilibria. While hidden attractors have the basins of attraction, which are not connected with equilibria, and their search requires the development of special analytical and numerical methods.



For various applications, the transition of system's state to a hidden attractor, caused by external disturbances, may result in undesirable behavior and is often the cause of accidents and catastrophes. For various engineering applications the importance of identifying hidden attractors is related with the classical problems of determining the boundaries of global stability in the space of parameters and in the phase space. Outer estimations of the global stability boundary in the space of parameters and the birth of self-excited oscillations in the phase space can be obtained by the linearization around equilibria and the analysis of local bifurcations and are related with various well-known conjectures on global stability by the first approximation (see, e.g. Andronov's proof of the conjecture on the Watt regulator global stability by the first approximation). Inner estimations of the global stability boundary can be obtained by classical sufficient criteria of global stability. In the gap between outer and inner estimations there is exact boundary of global stability which study requires the analysis of nonlocal bifurcations and hidden oscillations.

This lecture is devoted to well-known theoretical and practical problems in which hidden attractors (their absence or presence and disposition) play an important role.

### References

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2. Kuznetsov N.V., Theory of hidden oscillations and stability of control systems, **Journal of Computer and Systems Sciences International**, 59(5), 2020, 647-668.
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**Short Bio: Nikolay V. Kuznetsov** graduated from the St. Petersburg University, Russia in 2001. In 2004 he received the Candidate of Science degree and in 2016 the Doctor of Science degree from St. Petersburg University, where he is currently a Professor and the Head of the Department of Applied Cybernetics. From 2018, the research group chaired by Prof. Kuznetsov has the status of the Leading Scientific School (Center of Excellence) of Russia in the field of mathematics and mechanics. In 2020 he was named Professor of the Year in the field of mathematics and physics in Russia. In 2008, he defended his Ph.D. degree at the University of Jyväskylä (Finland), where now he is a Visiting Professor and co-chair of the Finnish-Russian Educational & Research program organized together with St. Petersburg University. In 2020 he was elected as foreign member of the Finnish Academy of Science and Letters. Since 2018, he is the Head of the Laboratory of information and control systems at the Institute for Problems in Mechanical Engineering of the Russian Academy of Science. Prof. Kuznetsov's research interests are in nonlinear dynamics and applied mathematics. In his works, a combination of rigorous analytical and reliable numerical methods allowed for both the advancement in solving previously known fundamental open problems as well as the development of modern engineering technologies.





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## Reliability and Maintenance

**Piero Baraldi** – *Predictive Maintenance in Industry 4.0*  
Politecnico di Milano, Italy

**Abstract:** As the digital, physical and human worlds continue to integrate, the 4th industrial revolution, the internet of things and big data, are changing our industrial world. In this fast-paced changing environment, in many industrial sectors maintenance is only starting its race to digitalization for increasing productivity, optimizing operating performance, reducing lifecycle costs, extending operating periods between maintenance and reducing downtimes, frequency and severity of unanticipated failures.

As maintenance is directly related with all industrial activities, improving maintenance is a key issue in the global economy. Inevitably, then, the use of big data and artificial intelligence for predictive maintenance has attracted great interest.

This workshop will present the specific challenges of predictive maintenance in Industry 4.0 with reference to detecting incipient failures, classifying their causes, predicting the system Remaining Useful Life (RUL), with its corresponding uncertainty. By way of examples of application, the main challenges towards the deployment of predictive maintenance and their effective integration in the operation and maintenance of industrial systems will be discussed.

**Eric Chatelet** – *Maintenance Optimization of Wind Farms*  
Univerisité de technologie de Troyes, France

**Abstract:** The renewable energy sources are more and more developed nowadays in order to reduce our carbon footprint. The maintenance of the renewable energy production systems has to be taken into account to improve their lifetime and life cycle costs (and also their carbon footprint). In order to optimize this maintenance process, it needs to develop adapted modelling able to assess reliability of these systems taking into account costs and dynamic effects (due to the wind speed variations, etc.). Among the reliability models, the UGF (Universal Generating Function) models are particularly adapted with a fast calculation time, and consequently interesting in optimization processes. Additionally, they are able to take into account dependencies (influences between components) and mixed with other models as Markov processes. Alternative simulation models can be also used as Petri nets (briefly presented), but if they assess efficiently the reliability of complex systems, the associated computing times are too expensive to be exploited in optimization processes. Some other correlated problems will be shortly discussed in the last part of the presentation (topology of wind farms, economical lifetime and the repowering problem).



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## Mathematical Modelling and Numerical Simulations

**Ulrich Rüde** – *Scalable Multiphysics Simulations*  
CERFACS and Universitaet Eflangen-Nuerberg, Germany

**Abstract:** Many modern simulation tasks arise from so-called multiphysics problems that require the coupling of individual simulation methods. This arises e.g. from the direct numerical simulation of multiphase flows. Here the hydrodynamics may be represented by an Eulerian model while a solid particulate phase is represented in a Lagrangian framework. Massively parallel simulations must then employ scalable flow solvers plus a scalable parallel integration method for computing the evolution of the particle trajectories. Additionally, however, these two frameworks must cooperate in parallel without losing scalability.

In the talk we will highlight some challenges arising and will report on the methods that we have developed.

## Graph Colorings and Coverability of Graphs

**Riste Škrekovski**- *Some Results on Unique-maximum Coloring of Plane Graphs*  
Faculty of Information Studies, Novo mesto & Faculty of Mathematics and Physics,  
University of Ljubljana, Slovenia

**Abstract:** A unique-maximum coloring of a plane graph  $G$  is a proper vertex coloring by natural numbers such that each face  $\alpha$  of  $G$  satisfies the property: the maximal color that appears on  $\alpha$ , appears precisely on one vertex of  $\alpha$  (or shortly, the maximal color on a face is unique on that face). Fabrici and Göring proved that six colors are enough for any plane graph and conjectured that four colors suffice. Thus, this conjecture is a strengthening of the Four Color Theorem. Wendland later decreased the upper bound from six to five.

We first show that the conjecture holds for various subclasses of planar graphs but then we disprove it for planar graphs in general. Thus, the facial unique-maximum chromatic number of the sphere is not four but five. In the second part of the talk, we will consider various new directions and open problems.

Joint work with Vesna Andova, Bernard Lidický, Borut Lužar, and Kacy Messerschmidt.

**Borut Lužar** – *Edge-coloring Subcubic Graphs with Five Colors*  
Faculty of Information Studies, Novo Mesto, Slovenia

**Abstract:** A proper edge-coloring of a graph is an assignment of colors to its edges such that adjacent edges receive distinct colors. From Vizing's theorem it follows that every graph with maximum degree 3 admits a proper edge-coloring using at most 4 colors. It turns out that by allowing just one additional color (i.e., using 5 colors), we are able to add many interesting constraints to our edge-coloring setting and still be able to complete the coloring. In the talk, we will survey results about the most studied edge-coloring variations; we will begin with a recent result on the strong edge-coloring, and continue by considering normal and adjacent vertex-distinguishing edge-colorings. Although a considerable amount of research was done on the mentioned topics, there are still many interesting open problems waiting to be solved.



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**Kenny Štorgel** – *Further Extensions of the Grötzsch Theorem*  
Faculty of Information Studies, Novo Mesto, Slovenia

**Abstract:** The Grötzsch Theorem states that every triangle-free planar graph  $G$  admits a proper 3-coloring, i.e. a coloring of the vertices of  $G$  with three colors such that adjacent vertices are assigned distinct colors. However, we may also allow triangles in general planar graphs and still retain 3-colorability. Havel conjectured that a 3-colorable planar graph may contain arbitrarily many triangles as long as they are sufficiently far apart. This conjecture was proved by Dvořák, Král', and Thomas. On the other hand, there are 3-colorable planar graphs that may have close triangles (even incident). A result by Dross et al. states that every planar graph obtained as a subgraph of the medial graph of a bipartite plane graph is 3-colorable.

As mentioned, the Grötzsch Theorem has many generalizations, although, perhaps the most well-known is a result of Grünbaum and Aksenov, giving 3-colorability of planar graphs with at most three triangles, which is in general best possible. A lot of attention was also given to extending 3-colorings of subgraphs of triangle-free planar graphs to the whole graph. In particular, a result of Aksenov, Borodin, and Glebov states that we can precolor any two non-adjacent vertices in a triangle-free planar graph and retain 3-colorability. Furthermore, several other results exist which deal with precolorings of a face of certain length in a triangle-free planar graph.

In this talk, we will present the above-mentioned results and provide further extensions of the Grötzsch Theorem by considering 3-colorings of planar graphs with at most one triangle. In particular, we show that a precoloring of any two non-adjacent vertices and a precoloring of a face of length at most 4 can be extended to a 3-coloring of the whole graph. Additionally, we show that for every vertex of degree at most 3 in a planar graph with at most one triangle, a precoloring of its neighborhood with the same color extends to a 3-coloring of the whole graph. The latter result yields an affirmative answer to a conjecture on adynamic coloring.

This is a joint work with Hoang La and Borut Lužar.

**Mirko Petruševski** – *Coverability by Parity Regular Subgraphs*  
Faculty of Mechanical Engineering, Ss. Cyril and Methodius University, Skopje, Republic of North Macedonia

**Abstract:** Only two types of graphs are 'parity regular', that is, have all their vertex degrees of the same parity. Namely, these are the 'even graphs' and the 'odd graphs', where a graph is said to be even (resp. odd) if all its vertex degrees are even (resp. odd). An old result of Matthews from 1978 establishes a connection between nowhere-zero  $2^k$ -flows in graphs and coverability by  $k$  even subgraphs, which in view of Jaeger's 8-flow and 4-flow theorems, respectively, implies that every bridgeless graph admits a cover by three even subgraphs and that every 4-edge-connected graph is coverable by two even subgraphs.

In this talk, we present some new results that complement the aforementioned. In particular, we characterize coverability by three odd subgraphs, and discuss coverability by every other possible combination of three parity regular subgraphs.

This is a joint work with Riste Škrekovski.