



# Workshop on Applied Mathematics - Discrete Mathematics Seminar

Time: Tuesday, 10th September 2019, 15<sup>00</sup>-18<sup>00</sup> Abstracts:

Venue: EA553, VŠB – TU Ostrava

Free Registration:

<https://forms.gle/Shg5dJbDJhxegT2v5>

Program:

Chairman: Petr Kovář

15<sup>00</sup> - 15<sup>50</sup> **Martin Škoviera** - *Point-line configurations in projective spaces and the structure of cubic graphs*

16<sup>00</sup> - 16<sup>50</sup> **Edita Máčajová** - *Smallest counterexample to the Fulkerson conjecture must be cyclically 5-edge-connected*

17<sup>00</sup> - 17<sup>30</sup> coffee break

17<sup>30</sup> - 17<sup>45</sup> **Tom Raiman** - Use of generalized truncation for computer search for  $(k,g)$ -graphs

17<sup>45</sup> - 18<sup>00</sup> **Petr Kovář** - *Supermagic graphs with arbitrary degree differences*

**Martin Škoviera** - *Point-line configurations in projective spaces and the structure of cubic graphs*

We propose a unifying approach to several important conjectures in graph theory in terms of nowhere-zero flows on cubic graphs where flow-values are points of a configuration of lines in a projective space and outflow-patterns are triples of points sharing a line of the configuration. We then focus on the conjecture of Claude Berge suggesting that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. It turns out that cubic graphs that require five (or more) perfect matchings to cover their edges are extremely rare and hence difficult to find. We show that they can be efficiently studied by means of flows whose outflow patterns form a configuration of six lines spanned by four points of the 3-dimensional projective geometry  $PG(3,2)$  in general position. We employ this knowledge to provide a great variety of constructions of cubic graphs that cannot be covered with four perfect matchings. We finally show that one of our constructions provides counterexamples to a conjecture of Brinkmann et al. about the length of a shortest cycle cover of a cyclically 4-edge-connected cubic graph.

This is a joint work with Edita Máčajová.

**Edita Máčajová** - *Smallest counterexample to the Fulkerson conjecture must be cyclically 5-edge-connected*

The Fulkerson conjecture belongs to one of the most prominent open problems in Graph theory. It suggests that the edges of any bridgeless cubic graph can be covered with six perfect matchings in such a way that each edge belongs to exactly two of them. The origin of this conjecture lies in mathematical programming and the conjecture itself has close connections to configurations of points and lines in the projective space. Despite the fact that Berge and Fulkerson made this conjecture almost half a century ago, it has been verified only for several explicitly defined families of graphs. During the talk we will present the current state of this conjecture as well as its connections to other important conjectures in graph theory. As the main result we show that Fulkerson's conjecture can be reduced to cyclically 5-edge-connected cubic graphs.

This is a joint work with Giuseppe Mazzuocolo.