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Group distance magic labelings of hypercubes and Cartesian products of cycles

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Let G be a graph with n vertices and f a bijection $f : V(G) \to \{1, 2, \ldots, n\}$. We define the *weight* of vertex x as the sum of the labels of its neighbors, that is,

$$w(x) = \sum_{xy \in E(G)} f(y).$$

When all vertices have the same weight, say $w(x) = m$ for every x, then f is called *distance magic labeling* and G is a *distance magic graph*.
However, it turns out that this type of magic labeling is very restrictive and consequently even many classes of vertex transitive graphs are not distance magic.
As an example, we prove that for $d \equiv 0, 1, 3 \pmod 4$ the hypercube Q_d with 2^d vertices is not distance magic. On the other hand, we disprove a conjecture by Acharya, Rao, Singh and Parameswaran, who believed that hypercubes are not distance magic except Q_2 and present a distance magic labeling for Q_6. This was recently generalized by Gregor and Kovar who found a distance magic labeling for of Q_d for any $d \equiv 2 \pmod 4$.
Such negative results then rise a question whether it would not be more natural to perform the addition in Z_n rather than in $\mathbb{Z}$. Graphs that satisfy the above definition with the provision that the addition is performed in Z_n will be called Z_n-distance magic.
To support this idea, we show some examples of graphs that are not distance magic yet are Z_n-distance magic. We show that when we perform addition Z_{2^d} rather than in $\mathbb{Z}$, then Q_d is Z_{2^d}-distance magic if and only if d is even.
We present some results on Γ-distance magic labelings of products of cycles and pose several open problems.