Supermagic graphs with many different degrees

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A graph $G = (V, E)$ is called *supermagic* if there exists a bijection $f : E \to \{1, 2, \ldots, |E|\}$ called *supermagic labeling* such the *weight* of every vertex $x \in V$ defined as the sum of labels $f(xy)$ of all edges xy incident with x is equal to the same number m, called the *supermagic constant*. That is,

$$\exists m \in \mathbb{N}\ \forall x \in V : w(x) = \sum_{xy \in E} f(xy) = m.$$

Typically, the classes of graphs studied in this context are vertex-regular or even vertex-transitive. Recently, Kovář et al. affirmatively answered a question by Madaras:
Does there exist a supermagic graph with k different degrees for any positive integer k?
Because their construction provided only graphs where all degrees were even, they asked the following more specific question:
Does there exist a supermagic graph with k different odd degrees for any positive integer k?
We answer this question in the affirmative by providing a construction based on the use of 3-dimensional magic rectangles.
The obvious next step is to ask the following:
Let $k > 1$ be an integer and $s_1, s_2, \ldots, s_{k-1}$ a sequence of positive integers. Is there a positive integer a_1 such that there exists a supermagic graph with degrees $a_1, a_2, \ldots, a_k$ where $a_{i+1} = a_i + s_i$ for $i = 1, 2, \ldots, k-1$?
We also present a sketch of two proposed constructions of graphs hopefully answering the above question in the affirmative.

Keywords: Supermagic graphs, magic-type labeling, edge labeling