G-designs for some small graphs

Dalibor Fronček

University of Minnesota Duluth

(joint work with with Danny Banegas and Aidan Carlson)

středa 18. června 2025,   od 13:00,   EB226

Almost exactly three years ago, I gave a talk at DiMaS with the following abstract:

Decomposition of complete graphs
unicyclic graphs with seven edges

An G-decomposition of a graph H is a collection of subgraphs $G_1, G_2, \ldots, G_s$ of H, all isomorphic to G, such that every edge of H belongs to exactly one copy G_i. When $H = K_n$, the decomposition is also called a G-design. A unicyclic graph is a graph containing precisely one cycle.

The problem of decomposition of complete graphs has been completely solved for graphs with at most six edges, and with a few exceptions also for graphs with eight edges. However, for graphs with seven edges it was almost untouched except for trees. We therefore take the next natural step and examine decompositions into unicyclic graphs with seven edges.

The obvious necessary condition for such a decomposition into graphs with seven edges is that $n \equiv 0, 1 \pmod 7$. We show that the condition is also sufficient for any connected unicyclic bipartite graph H with seven edges when $n \equiv 0 \pmod 7$ and $n \equiv 1 \pmod{14}$ except for several cases when $n = 7$ or 8. We may also show some partial results for the non-bipartite case.

For disconnected graphs, we present just partial results for the bipartite case, as the work is still in progress.

This [was] a joint work with Michael Kubesa.

This time I will report on the progress made mostly in collaboration with my students. In particular, I will focus on disconnected unicyclic bipartite graphs with seven edges and nine or ten vertices and present several structurally different construction methods.