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# Cycles and perfect matchings in snarks

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Every 3-edge-colourable cubic graph has a set of three perfect matchings that cover all of its edges. Conversely, if the edge set of a cubic graph can be covered with three perfect matchings, the matchings must be disjoint and the graph 3-edge-colourable. The cubic graphs not possesing any regular 3-edge-colourings are called *snarks*. It follows in snarks any collection of three perfect matchings misses some edges of a graph. The minimum number of edges of a cubic graph $G$ left uncovered by any three perfect matchings will be called the *colouring defect* (shortly *defect*) of $G$ and will be denoted by $\mathrm{df}(G)$. Clearly, a cubic graph has defect zero if and only if it is 3-edge-colourable and the number is grater than zero for snarks. This invariant has been introduced by E. Steffen in 2015 [3] and it appears to be interesting invariant of snarks.

We try to exhibit relationships of the defect of a snark to other invariants such as (colouring) *resistance*, *oddness*, *girth*, *criticality*, *irreducibility* [2] and others. It is quite easy to show that defect of snarks is greater or equal to three. Far the most of the snarks have defect equal to three, which implies a strong structural constraint – the existence of a six-cycle. We will show the smallest snark with defect greater than 3 and discuss the consequences of this observation. We will set the colouring defect for several families of snarks or, at least, we will present the working hypotheses on them. We will also exhibit the close relationship of the study of snarks with small defect and well-known hypotheses e.g. Berge-Fulkerson Conjecture and 6-decomposition theorem [1, 2] (Jaeger Conjecture).

# Reference


[1] J. Karabáš, E. Máčajová, R. Nedela, 6-decomposition of snarks, *European J. Combin.* **34** (2013), 111–122.

[2] R. Nedela and M. Škoviera, Decompositions and reductions of snarks, *J. Graph Theory* **22** (1996), 253–279

[3] E. Steffen, 1-Factor and cycle covers of cubic graphs, *J. Graph Theory* **78** (2015), 195–206