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# Symmetric chain decompositions and the central levels problem

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A symmetric chain in the  $n$ -dimensional cube  $Q_n$  is a path of length  $n - 2k$  that starts on level  $k$  and ends on level  $n - k$  for some  $k$ . Greene and Kleitman in 1970's described a decomposition of  $V(Q_n)$  into symmetric chains. The central levels problem asserts that the subgraph of the  $(2m + 1)$ -dimensional cube induced by all vertices between levels  $m + 1 - l$  and  $m + l$  (i.e. the central  $2l$  levels) has a Hamilton cycle for all  $m \geq 1$  and  $1 \leq l \leq m + 1$ . This problem raised by Savage is a common generalization of the well-known middle levels problem and classical binary Gray codes.

In the first part we give introduction to symmetric chain decompositions (SCDs) and Gray codes with their applications. In the second part we present a constructive solution to the central levels problem that is based on the Greene-Kleitman SCDs. In particular, we show that the Greene-Kleitman SCD extends to a Hamilton cycle of  $Q_n$  and there is a loopless algorithm to generate it. This is a joint work with Torsten Mütze and Ondřej Mička.