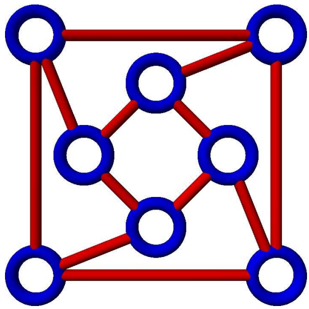

CSGT 2024
Abstracts

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The conference is partially supported by the Faculty of Electrical Engineering and Computer Science, VSB – Technical University of Ostrava.

VERTEX-TRANSITIVE NUMBERS OF GRAPHS

Martin Bachratý*, Štefánia Glevitzká

A graph is vertex-transitive if its automorphism group acts transitively on its vertices. A vertex-transitive closure of a graph G is any vertex-transitive graph that contains G as its spanning subgraph. The vertex-transitive number of G is the smallest possible degree of its vertex-transitive closure. In this talk we will explain motivation for studying this relatively new concept. Furthermore, we will formulate various interesting questions and in some cases also provide (at least partial) answers to them.

The first author acknowledges support from the APVV Research Grants 19-0308 and 22-0005, and from the VEGA Research Grants 1/0567/22 and 1/0069/23.

TWIN-WIDTH AND FEEDBACK EDGE NUMBER

Jakub Balabán*, Robert Ganian,
Mathis Rocton

Twin-width is a graph parameter introduced in 2020 by Bonnet et al., and it has attracted a lot of attention since then. It has been shown that FO model checking is FPT when parameterized by twin-width (given a certificate of small twin-width as part of the input). At the same time, many graph classes have bounded twin-width, including planar graphs and graphs of bounded treewidth. It is easy to see that twin-width is in $\mathcal{O}(k)$ when k is the feedback edge number, i.e., the number of edges one needs to remove to destroy all cycles. In this talk, we show how to improve this bound to $\mathcal{O}(\sqrt{k})$, which is asymptotically tight. We also mention a few related results, namely an FPT algorithm for recognizing graphs of twin-width 2 and an FPT approximation of twin-width with a constant additive error, both parameterized by the feedback edge number.

EQUIANGULAR LINES AND REGULAR GRAPHS

Igor Balla

In 1973, Lemmens and Seidel posed the problem of determining the maximum number of equiangular lines in $\mathbb{R}^r$ with angle $\arccos(\alpha)$ and gave a partial answer in the regime $r \leq 1/\alpha^2 - 2$. At the other extreme where r is at least exponential in $1/\alpha^2$, recent breakthroughs using graph-theoretic ideas have almost completely resolved this problem.

In this talk, we will discuss how orthogonal projection of matrices can be used in order to unify and improve upon all previous approaches, thereby yielding bounds which bridge the gap between the aforementioned regimes and are best possible either exactly or up to a small multiplicative constant. As a byproduct, we obtain the first extension of the celebrated Alon–Boppana theorem to dense graphs, with equality for strongly regular graphs corresponding to families of $\binom{r+1}{2}$ equiangular lines in $\mathbb{R}^r$.

WEIGHTED p-MEDIAN AND WEIGHTED p-CENTER: ANALYSIS OF SENSITIVITY OF OPTIMAL SOLUTION

Peter Czimmermann*

Weighted p-median and weighted p-center in graphs are well-known location problems with many practical applications.
In our contribution, we focus on sensitivity analysis of the optimal solution of the mentioned location problems concerning changes in the conditions in the graph. We consider the following cases:
1) Elongation of the length of the edge or the pair of edges. (In practice, it can represent the elongation of travel time of a road segment.)
2) Elongation of all edges. (In practice, this corresponds to an extension of driving time during rush hour.)
3) Changes of weights of vertices. (Practically, it represents the movement of residents due to work, school, shopping, etc.)

This work was supported by the Slovak Research and Development Agency under Contract No. APVV-19-0441 "Allocation of limited resources to public service systems with conflicting quality criteria".

THEORETICAL AND COMPUTATIONAL APPROACH TO (k, g)-SPECTRA

Leonard Chidiebere Eze*, Dominika Mihálová, Robert Jajcay, and Tatiana Jajcayová

For each pair of parameters (k, g), where $k \geq 3$ and $g \geq 3$, the complete set of possible orders of connected k-regular graphs with girth g is referred to as *the spectrum of orders of* (k, g)*-graphs*, or *the* (k, g)*-spectrum*. Finding the (k, g)-spectrum for a specific pair of parameters (k, g) is extremely difficult, as it requires determining the minimum order $n(k, g)$ of the smallest connected k-regular graph with girth g. In this talk, we present some theoretical results on determining (k, g)-spectra together with some algorithms for generating (k, g)-spectra. For some specific parameter pairs, we explicitly list the (k, g)-spectra that we have determined and the associated algorithms.

This research is supported by VEGA 1/0437/23 and APVV-19-0308 Grants

UNIQUE-MAXIMUM COLORINGS OF PLANE GRAPHS

Igor Fabrici*, Simona Rindošová

A proper vertex coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices is used exactly once. Wendland (2016) proved that every plane graph is UM 5-vertex-colorable and Lidický, Messerschmidt, and Škrekovski (2018) constructed a plane graph with corresponding chromatic number 5.

In this talk, we consider following two modifications of the previous coloring:

1) a proper vertex-face coloring of a plane graph is unique-maximum (UM) if, for every face, the maximum color on its vertices and on the face itself is used exactly once,

2) a proper vertex coloring of a plane graph is unique-double-maximum (U2M) if, for every face, the highest color and the second highest color on its vertices are used exactly once,

compare these two colorings, and present upper bounds on the corresponding chromatic numbers for the set of plane graphs.

ON 1-2-3-CHROMATIC NUMBER

Andrea Feňovčíková

Let $G = (V, E)$ be a graph with no component isomorphic to K_2 and let $f : E \to \{1, 2, 3\}$ be an edge labeling of G. The weight of a vertex is defined as the sum of labels of all edges incident with that vertex. If the weights of any two adjacent vertices are distinct then f is called a chromatic 1-2-3 labeling of G. In this case the vertex weights give a proper vertex coloring of G where the color of a vertex is its vertex weight. This naturally leads to the concept of the 1-2-3-chromatic number. The 1-2-3-chromatic number is defined as the minimum number of colors taken over all colorings of G induced by chromatic 1-2-3 labelings of G.

In this talk, we present several basic results on this new parameter.

SEMI-MAGIC SQUARES OVER DIHEDRAL GROUPS

DALIBOR FRONČEK

Magic squares are among oldest known combinatorial objects, dating back to the 4-th century BC. A *magic square* of side n, $MS(n)$, is an $n \times n$ array with entries $1, 2, \ldots, n^2$, each appearing exactly once, such that every row, column, and the main forward and backward diagonals have the same sum $m = n(n^2 + 1)/2$, called the *magic constant*. A *semi-magic square* only requires the row and column sums to be equal.

Last year in Bardejovské Kúpele I presented results on dihedral supermagic labelings of some 4-regular graphs and observed that the results can be rather easily extended in certain classes of $4k$-regular graphs. Of the remaining regularities, the odd ones seem to be difficult, so I looked into $(4k + 2)$-regular graphs (with no success so far).

Since supermagic labelings of regular complete bipartite graphs are equivalent to semi-magic squares, I got soon attracted by constructions of *dihedral semi-magic squares* $SMS_{D_k}(n)$, where the entries are elements of D_k rather than integers. Obviously, for odd n such squares do not exist because D_k is of order $2k$.

Not too surprisingly I was able to find such rectangles of side $n \equiv 0 \pmod 4$, but not $n \equiv 2 \pmod 4$. I will present some constructions of $SMS_{D_k}(n)$ where $n \equiv 0 \pmod 4$ and $n^2 = 2k$.

APPROXIMATE CYCLE DOUBLE COVER

Babak Ghanbari* and Robert Šámal

The cycle double cover conjecture states that for every bridgeless graph G, there exists a family $\mathcal{F}$ of cycles such that each edge of the graph is contained in exactly two members of $\mathcal{F}$. Given an embedding of a graph G, an edge e is called a **bad edge** if it is visited twice by the boundary of one face. CDC conjecture is equivalent to bridgeless cubic graphs having an embedding with no bad edge. In this talk, we introduce non-trivial upper bounds on the minimum number of bad edges in an embedding of a cubic graph. We also introduce better upper bounds on the minimum number of bad edges in an embedding of cyclically $2k$-edge-connected and cyclically k-edge-connected cubic graphs. Finally, we introduce another approach to the CDC conjecture. Every embedding allows us to make a dual graph. The embedding gives a CDC if the dual graph has no loop. We are studying how to modify the embedding of a cubic graph by doing controlled modifications of the dual graph. The goal of that is to find a dual with a small number, or ideally no loop edge.

ON A RELATION BETWEEN G-GRAPHS AND LIFTS

Štefan Gyürki*, Pavol Jánoš, Jozef Širáň

Constructions of regular graphs based on groups are quite popular, since some of the graph invariants can be computed algebraically simply from the knowledge of the used group and its basic properties. The most famous such construction of graphs from groups are known as Cayley graphs. A similar construction of graphs having highly regular properties is called G-graphs. Another one is called the technique of voltage assignments, or lifts, which can be regarded, in a sense, as a generalization of the concept of Cayley graphs. An advantage of the lifting construction and G-graphs in comparison with the Cayley graphs might be that they can also produce graphs that are not regular.

In this talk, we will examine when they are giving regular graphs with prescribed girth and compare these two constructions. We provide sufficient conditions under which a G-graph can be obtained as a voltage assignment. In addition, we generalize two families of graphs important in the degree-girth problem which were constructed as G-graphs, after understanding their description as voltage assignments, for richer families.

We acknowledge the support from the Slovak Research and Development Agency under the grants APVV 19-0308, APVV 22-0005, and the Scientific Grant Agency of the Slovak Republic under the grants VEGA 1/0567/22, and VEGA 1/0069/23.

EDGE- AND VERTEX-GIRTH-REGULAR GRAPHS AND THEIR ROLE IN EXTREMAL GRAPH THEORY

Robert Jajcay

The concepts of edge- and vertex-girth-regular graphs stem from two intersecting areas of interest: Edge- and vertex-transitive graphs and graphs extremal with regard to the well-known Degree/Diameter and Cage Problems.

We call a k-regular graph of girth g *edge-girth-regular* if each of its *edges* belongs to the same number λ of girth cycles, and we call a k-regular graph of girth g *vertex-girth-regular* if each of its *vertices* is included in the same number (denoted by λ again) of girth cycles. Clearly, every edge-transitive graph is edge-girth-regular (and also vertex-girth-regular) and each vertex-transitive graph is vertex-girth-regular. Moreover, all the Moore graphs are necessarily edge-girth-regular; thereby connecting the topic to the Extremal Graph Theory problems mentioned above. This suggests that the study of these two classes of graphs might potentially lead to useful insights in both of the above-mentioned areas of Graph Theory.

In our talk, we shall address a variety of questions concerning edge- and vertex-girth-regular graphs. First of all, for both classes, we derive upper bounds on the parameter λ viewed as a function of the degree and girth of the considered graphs. This naturally leads to existence problems in which we ask about the existence of graphs with permissible parameters (k, g, λ). With regard to this question, we will present infinite classes of admissible parameter triples for which we can show that graphs with these parameters do not exist, as well as a number of constructions in the other case. Ultimately, in the case when the existence question can be answered in positive, in line with the Cage Problem, we address the question of the minimal order of a (k, g, λ) edge- or vertex-girth-regular graph. We derive some lower bounds for these orders as functions of the parameters, and present some settled cases in which we know the exact answers (sometimes relying on the use of computers).

$\mathcal{H}$-CLIQUE-WIDTH

PETR HLINĚNÝ, JAN JEDELSKÝ*

We generalize clique-width by defining $\mathcal{H}$-clique-width. We discuss its basic properties, its relation to planar product structure, and first-order transductions thereof.

We say that H is a *loop graph* if the vertex set of H is finite and H has no parallel edges. Let $H \in \mathcal{H}$ be a loop graph, and let $k \in \mathbb{N}$ be natural number. We say that ϕ is an (H, k)-expressing if ϕ uses the following operations to produces a graph $G = \phi()$ where every vertex $v \in V(G)$ is assigned a color $c_v \in [k]$ and a parameter vertex $p_v \in V(H)$:

- create_vertex(c, p), creating a single vertex graph whose unique vertex has color $c \in [k]$ and parameter vertex $p \in V(H)$;
- disjoint_union(ψ_1, ψ_2), creating a disjoint union of graphs $G_1 = \psi_1()$ and $G_2 = \psi_2()$
- add_edges($\psi_1, c_1 \neq c_2$), adding edges between all pairs of vertices u, v of $G_1 = \psi_1()$ satisfying all of the following conditions:
  - color of u is c_1
  - color of v is c_2
  - parameter vertices p_u of u and p_v of v are adjacent in H.
- recolor($\psi_1, c_1 \neq c_2$), recoloring all vertices of $G_1 = \psi_1()$ whose color is c_1 to c_2 (without changing their parameter vertices).

Let $\mathcal{H}$ be a class of loop graphs. We say that a simple graph G has *$\mathcal{H}$-clique-width* at most k if there exists a loop graph $H \in \mathcal{H}$ and (H, k)-expression ϕ such that $G = \phi()$.

CRITICALITY IN SPERNER'S LEMMA

Tomáš Kaiser*, Matěj Stehlík,
Riste Škrekovski

Sperner's lemma states that if a labelling of the vertices of a triangulation K of the d-simplex Δ^d with labels $1, 2, \ldots, d+1$ has the property that (i) each vertex of Δ^d receives a distinct label, and (ii) any vertex lying in a face of Δ^d has the same label as one of the vertices of that face, then there exists a rainbow facet (a facet whose vertices have pairwise distinct labels).

Tibor Gallai asked in 1969 whether Sperner's Lemma is 'critical' in the sense that for every triangulation K as above and every facet σ of K, there is a labelling satisfying (i) and (ii) such that σ is the unique rainbow facet. (The question is included as Problem 9.14 in Jensen and Toft's collection *Graph Coloring Problems*.)

In this talk, we show that the answer is affirmative for $d \leq 2$ (as already proved by Gallai). For every $d \geq 3$, however, we answer Gallai's question in the negative by constructing an infinite family of examples where no labelling with the requested property exists. The construction is based on the properties of a convex 4-polytope which had been used earlier to disprove a claim of Theodore Motzkin on neighbourly polytopes.

Joint work with Matěj Stehlík and Riste Škrekovski.

THREE-CUTS ARE A CHARM: ACYCLICITY IN 3-CONNECTED CUBIC GRAPHS

František Kardoš*, Edita Máčajová, Jean Paul Zerafa

In 2023, the three authors solved a conjecture (also known as the S_4-Conjecture) made by Mazzuoccolo in 2013: Let G be a bridgeless cubic graph. Then there exist two perfect matchings of G such that the complement of their union is a bipartite subgraph of G. This is a step closer to comprehend better the Fan–Raspaud Conjecture and eventually the Berge–Fulkerson Conjecture. The S_4-Conjecture, now a theorem, is also the weakest assertion in a series of three conjectures made by Mazzuoccolo in 2013, with the next stronger statement being: For every bridgeless cubic graph G there exist two perfect matchings of G such that the complement of their union is an acyclic subgraph of G. Unfortunately, this conjecture is not true: Jin, Steffen, and Mazzuoccolo later showed that there exists a counterexample admitting 2-cuts. Here we show that, despite of this, every cyclically 3-edge-connected cubic graph satisfies this second conjecture.

ALGORITHMS FOR H-FREE GRAPHS

Tereza Klimošová

Many fundamental graph problems are NP-hard in general but are known to be solvable in polynomial time on some classes of H-free graphs, that is, graphs without an induced copy of a fixed graph H.

The motivation for studying the complexity of NP-hard problems in this setting is twofold: One, to understand which structural properties make these problems hard, and two, to provide efficient algorithms for NP-hard problems on rich classes of graphs. Ideally, one would like to obtain a full dichotomy between the graphs H for which the problem is NP-hard on H-free graphs and the rest, for which we know a polynomial time algorithm on H-free graphs.

I will focus on two problems: Maximum Weight Independent Set and 3-Colouring. The aforementioned dichotomy is available for neither of the problems, despite significant research efforts. However, we have a good understanding of what the dichotomy should be. In my talk, I will survey results in this area.

ON ŠOLTÉS GRAPHS

Nino Bašič, Martin Knor*, Riste Škrekovski

Let G be a graph. Its Wiener index, $W(G)$, is the sum of all distances in G. In 1991 Šoltés observed that $W(C_{11}) = W(P_{10})$. That means that if you delete any vertex from the cycle C_{11}, the Wiener index remains the same. Currently, graphs with this property are called Šoltés graphs, and the only Šoltés graph known to this day is C_{11}. In the talk we summarize some recent results on this topic.

This research was supported by the APVV Research Grants 19-0308 and 22-0005 and the VEGA Research Grants 1/0567/22 and 1/0069/23.

CLEVER COUNTING OF THE NUMBER OF CERTAIN DISCRETE STRUCTURES

Tatiana Jajcayová, Róbert Jajcay, Pavol Kollár*

Cayley Graphs are an extensively studied class of graphs, but many graphs, even those that are vertex-transitive, are not Cayley. Ginette Gauyacq's paper "On quasi-Cayley graphs" introduces a related structure, the quasi-Cayley graph. She created these as Cayley graphs over a quasi-groups, whose multiplication tables are Latin Squares, which she called "regular families".

Róbert Jajcay and Gareth Jones in their paper "r-regular families of graph automorphisms" generalised the aforementioned structure into "r-regular family" in order to measure how far a given vertex-transitive graph is from a Cayley graph. With the help of the computer power, our current research is aimed at the generation and enumeration of these r-regular families and also to gain further insights about these unexplored structures.

In this talk, we will examine several variations of an algorithm for upper bounds on the total number of r-regular families for a fixed number of permuted elements in feasible computation time. This method of counting could turn out to be much more general-purpose and usable in other discrete enumeration problems.

Financované EÚ NextGenerationEU prostredníctvom Plánu obnovy a odolnosti SR v rámci projektu č. 09I03-03-V02-00036

GENERALIZED LABELLINGS OF CIRCULANT GRAPHS

Přemysl Holub, Martin Kopřiva*

By an $L(3,2,1)$-labelling of a graph G we mean an assignment of non-negative integers to vertices G according to the following rules: adjacent vertices are labelled by values differing by at least 3, vertices at distance two apart are labelled by values differing by at least 2 and vertices at distance three are labelled by values differing by at least 1. It is natural to ask for the smallest positive integer, denoted by $\lambda_{(3,2,1)}(G)$, such that G admits an $L(3,2,1)$-labelling with span $\lambda_{(3,2,1)}(G)$, specifically with values $0, \ldots, \lambda_{(3,2,1)}(G)$.

Let $S \subset [n-1]$, where $n \in \mathbb{N}, n \geq 3$, such that if $x \in S$, then also $(n-x) \in S$. A graph $G = (V(G), E(G))$, where $V(G) = \{u_1, \ldots, u_n\}$ and $E(G) = \{u_i u_j ; |u_j - u_i| \in S, i, j \in \{1, \ldots, n\}\}$, is a *circulant* $C_n(S)$ generated by the set S.

In this talk we investigate $L(3,2,1)$-labelling of some circulant graphs. We consider two types of circulants: dense ($|S| \geq \frac{n}{2}$) and sparse ($|S| = 3$ or $|S| = 4$). We establish some exact values and bounds for $\lambda_{(3,2,1)}(G)$ for these circulants.

COVERING COMPLETE GRAPHS BY K_3 AND K_4

Petr Kovář*, Yifan Zhang

In a paper by D. Lukáš, P. Kovář, T. Kovářová, and M. Merta, "A parallel fast boundary element method using cyclic graph decompositions" from 2015 the authors proposed a method for decomposing calculations of a dense matrix into submatrices using a cyclic decomposition of complete graph K_n into n complete subgraphs K_k. The parameters n and k of the cyclic decomposition are not independent, since $n = k^2 - k + 1$. In addition, the existence of cyclic decomposition is based on the so-called ρ-labelings, the existence of which is known only if $k - 1$ is a power of a prime number. Also, using cyclic decompositions limits the approach to systems with shared memory.

Due to the development of supercomputer architecture, the possibility of implementation using fast local memories of individual cores is emerging, however the size of the local memory is limited. Therefore it makes sense to study decompositions of K_n into K_k, where k is a small fixed integer while n grows arbitrarily with the number of supercomputer cores. Such decompositions cannot be cyclic. The decompositions are based, for example, on Steiner triple systems or quadruple systems. Moreover, for many values of n, such decompositions do not exist.

On the other hand, instead of decompositions, graph coverings based on the constructions of combinatorial designs, can be successfully used. A small part of the calculation is duplicated. This can be modeled by duplicated edges, called excess. We show that for simultaneous covers by K_3 and K_4 exist for all values of n, and the size of the excess is constant and reasonably small. We answer almost completely the question of the existence of covers with the smallest possible number of covering graphs.

ON THE SEGMENT NUMBER OF A PLANAR GRAPH

Sabine Cornelsen, Giordano Da Lozzo,
Luca Grilli, Siddharth Gupta,
Jan Kratochvíl*, Alexander Wolff

The line cover number of a planar graph G is the minimum number of lines that support all the edges of a plane straight-line drawing of G. The segment number of G is the minimum number of connected straight-line segments that are formed by a plane straight-line drawing of G. We prove that the segment number is in FPT when parameterized by several standard invariants of the input graph. We find of particular interest that even the list-segment version of the problem remains Fixed Parameter Tractable.

Thanks to the Czech Science Foundation research grant GAČR 23-04949X

EDGE COLOURINGS OF CUBIC POLES

Robert Lukoťka*

Cubic graphs can be split into two classes: colorable graphs which admit a proper-3-edge colouring and uncolourable graphs which do not. Bridgeless cubic graphs that are not 3-edge-colourable, called snarks, are at the hearth of many important open problems like Cycle Double Cover Conjecture, Five Flow Conjecture, and Fulkerson Conjecture. To understand uncolourability of a snark one can take a cut containing k edges and split the graph into two graph parts, called cubic k-poles.

Each 3-edge-colouring of a k-pole induces a k-tuple of colours on the dangling edges, called boundary colourings. All colourings of the k-pole induce a (multi)set of boundary colourings, called colouring set. A colouring set contains only colourings satisfying parity lemma and the set has to be closed under Kempe switches. For $k \leq 5$ these two conditions are not only necessary but also sufficient. We will focus on the case where $k = 6$. We introduce a new equivalence relation that greatly reduces the number of colouring sets one needs to consider. We present the results of computational experiments using this equivalence relation.

If we restrict ourselves to planar graphs, then besides planarity requirement for Kempe switches, we have another tool that restricts colouring sets of k-poles, Four Colour Theorem. As an possible additional tool we will use flow polynomials. We show that certain generalised flow polynomials are an efficient tool to capture the number of colourings with given boundary. We explore conditions that Four Colour Theorem imposes on the polynomial and study k-poles that are close to "refuting" the Four Colour Theorem with respect to their polynomial coefficients.

ON ANTI-A-WALKS IN PLANE GRAPHS

Štefan Berežný, Tomáš Madaras*,
Daniela Matisová, Juraj Valiska

Given a graph G and its rotation system $\mathcal{R} = \bigcup_{v \in V(G)} R(v)$ (where $R(v)$ is a cyclic clockwise ordering of edges incident with v), a walk $(v_0, e_1, v_1, \ldots, e_k, v_k)$ in G is an *anti-A-walk* if, for every $i \in \{1, \ldots, k-1\}$, the edges e_i, e_{i+1} are not successive in $R(v_i)$. A particular example of such a walk is earlier known *cut-through trail* in a 4-regular plane graph (where the edges e_i, e_{i+1} are opposite in $R(v_i)$).

Focusing on study of properties of anti-A-walks in plane graphs, we address the question of their existence between each pair of vertices. While this is not always possible in 4-regular plane graphs (resp. plane graphs of minimum degree at least 4) in the sense of existence of cut-through paths, the problem is open (even with anti-A-walks) for plane graphs of minimum degree 5. We developed a set of tools (in Maple computer algebra system) for testing path anti-A-connectivity of 5-regular plane graphs; in addition, for finding the shortest anti-A-paths (resp. trails) between two vertices in a graph with given rotation system, we used integer linear programming approach. We also present large families of polyhedral graphs of minimum degree 5 (with exponentially many members for fixed number of vertices) in which every pair of vertices is connected by an anti-A-path.

CONFLICT-FREE COLORING OF PLANAR GRAPHS

Lukáš Málik

A conflict-free coloring of a graph is a coloring of vertices such that for every vertex there is a color that appears exactly once in its (open/closed) neighborhood. The smallest number of colors required to color a graph G in such a way is called the conflict-free chromatic number of G, denoted by $\chi_{CFC_c}(G)$ for closed neighborhood and $\chi_{CFC_o}(G)$ for open neighborhood. For a class of graphs $\mathcal{C}$ we define $\chi_{CFC}(\mathcal{C}) = max\{\chi_{CFC}(G) \mid G \in \mathcal{C}\}$. Motivated by the frequency assignment problem, this type of coloring was first introduced in a geometric setting by Even et al. in 2003.

It has been shown by Z. Abel et al. that for the class of planar graphs $\mathcal{P}$, the following holds: $3 \leq \chi_{CFC_c}(\mathcal{P}) \leq 4$. In this talk, we will present an improved lower bound achieving $\chi_{CFC_c}(\mathcal{P}) = 4$. Then we show a new way of looking at problems concerning mainly conflict-free coloring with respect to the open neighborhood and mention its relation to result by F. Huang et al. which proves that $\chi_{CFC_o}(\mathcal{P}) \leq 5$.

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TOWARDS THE SMALLEST SIGNED NON-4-COLOURABLE GRAPH

František Kardoš, Matúš Matok*

In 2016, Máčajová, Raspaud and Škoviera conjectured that all signed planar graphs are signed 4-colourable. Three years later, Kardoš and Narboni found a counterexample, after having translated the problem of signed 4-colourablity of a planar triangulation into an equivalent problem in the dual – the existence of a so-called consistent semi-2-factor (i.e., a collection of disjoint cycles covering all positive vertices, with every cycle containing an even number of them) in a vertex signed dual 3-connected cubic planar graph.

We study the properties of 3-edge-cuts and 4-edge-cuts with respect to consistent semi-2-factors in order to establish structural properties of a smallest counterexample. We classify 3-poles (graphs obtained by cutting along 3-edge-cuts) according to those properties into twelve classes, with the goal of searching exhaustively for a smallest representative of each class. We have developed a program that generates cubic planar graphs with specified structure (avoiding reducible configurations) and that verifies the existence of specific semi-2-factors. We have found the smallest representatives for six classes out of the twelve so far. The rest remains an open problem.

TWO-FACTORIZATIONS OF SOME REGULAR GRAPHS

Mariusz Meszka

A *k-factor* in a graph G is a k-regular spanning subgraph of G. A *k-factorization* of G is a collection $\{F_1, F_2, \ldots, F_t\}$ of edge-disjoint k-factors such each edge of G belongs to exactly one F_i. We say that G has an *F-factorization* if each F_i, $i = 1, 2, \ldots, t$, is isomorphic to F.

One of the best-known open problems concerning two-factorizations is the famous Oberwolfach problem, posed by G. Ringel in 1967, which asks whether, for any two-factor F, the complete graph K_n (when n is odd) or $K_n \setminus I$ (when n is even and I is a one-factor removed from K_n) admits an F-factorization. Several years later A. Rosa suggested the following extension of the Oberwolfach problem, the so-called Hamilton–Waterloo problem, which asks for the existence of a two-factorization of K_n or $K_n \setminus I$ (depending on the parity of n) in which r of its two-factors are isomorphic to a given two-factor R, and the remaining q two-factors are isomorphic to a given two-factor Q, for any admissible r and q.

Results related to both these problems will be widely discussed. Moreover, various algorithmic methods for constructing two-factorizations, together with their relationship to other combinatorial objects and applications, will be presented.

COHERENT PARTITIONS OF CUBIC GRAPHS

Roman Nedela*, Michaela Seifrtová,
Martin Škoviera

A partition $\{A, J\}$ of the vertex-set of a connected cubic graph G with n vertices is *coherent* if A induces a tree, and J induces a graph with at most one edge. By parity argument, J is independent if $n \equiv 2 \pmod 4$, and J induces a one-edge graph if $n \equiv 0 \pmod 4$. In 1975 Payan and Sakarovitch proved that a cyclically 4-connected cubic graph with $n \equiv 2 \pmod 4$ admits a coherent partition. In 2024 (Discrete Mathematics) we confirmed the existence of coherent partition for cyclically 4-connected cubic graph with $n \equiv 0 \pmod 4$, and for some well-defined families of cubic graphs of cyclic connectivity 3. Motivation for investigation of coherent partitions in cubic graphs is 3-fold. First, if G is admits a coherent partition $\{A, J\}$, then the general lower bound $\Phi(G) \geq \lceil \frac{n+2}{4} \rceil$ for the decycling number (feedback number) $\Phi(G)$ is achieved. Secondly, we proved that a cubic graph G which admits a coherent partition is upper-embeddable, and we have a conjecture that upper-embeddability of G is equivalent to existence of a coherent partition of G. Hence there is a close relation between coherent partitions and maximum genus of cubic graph. Finally, coherent partitions appear as a useful tool in constructions of Hamilton cycles, or Hamilton paths in cubic graphs belonging to certain important families of cubic graphs drawn on surfaces (including leapfrog fullerines or cubic Cayley graphs).

This research was partially supported by grant APVV-19-0308.

CUBIC GRAPHS THAT ARE FAR FROM BEING COVERABLE BY FOUR PERFECT MATCHINGS

Robert Lukoťka,
Makuochukwu Felix Oguagbaka*

Cubic graphs without bridges can be categorized in terms of their perfect matching index—the minimum number of perfect matchings needed to cover the edges of a bridgeless cubic graph. A conjecture of Berge suggests that, at most, five perfect matchings are needed to cover any such graph. Cubic graphs that need a minimum of five perfect matchings to cover their edges are very rare, and only a few infinite families of nontrivial examples of such graphs exist. Such graphs are of particular interest, as famous, longstanding open problems, including the cycle double cover conjecture, Tutte's 5-flow conjecture, and Berge–Fulkerson conjecture, can be reduced to them.

In our talk, we define the four perfect matching cover defect of a cubic graph as the minimum number of edges of the graph not covered by four perfect matchings. This parameter serves as a way to capture how far a cubic graph is from being coverable by four perfect matchings. We also introduce several other invariants that capture this notion. Additionally, we construct an infinite family of cubic graphs that are cyclically 4-edge-connected, have girth at least 5, and are far from being coverable by four perfect matchings with respect to the invariants. Our study shows that each member of the infinite family has the same value for each invariant. However, to examine the differences between the invariants, we present a cubic graph that differs from the members of the infinite family in that it has two different values across the invariants.

keywords: cubic graph, snark, flow, covering, cyclic connectivity, perfect matching

The second author acknowledges the support from the Comenius University Grant UK/3249/2024 and the Extraordinary Scholarship of the Faculty of Mathematics, Physics and Informatics, Comenius University in Bratislava.

ASYMMETRIC GRAPHS AND PARTIAL AUTOMORPHISMS

Ján Pastorek*, Tatiana Jajcayová

Most of the graphs are *asymmetric*, i.e., they only have a trivial automorphism group [1]. However, the relation between asymmetric graphs, regular graphs, and graphs with non-trivial automorphism group is challenging to delineate. Removing just a single node from the graphical regular representation yields an asymmetric graph [2]. There are minimal asymmetric graphs, where all vertex-induced graphs have non-trivial automorphisms [3]. Moreover, almost all regular graphs have only a trivial automorphism group [4], one of the smallest being the well-known Frucht graph.

Following up on the research of Jajcayova et al. [5], we investigate the symmetry level of the graphs. *Symmetry level* of a graph Γ, $S(\Gamma)$, is defined as the ratio between the largest rank of a non-trivial partial automorphism of a graph Γ and its order $|V(\Gamma)|$, that is, $S(\Gamma) = \frac{n-d}{n}$.

Currently, we have graphs between $\frac{3}{4}$ and $\frac{2}{3}$ symmetry levels on graphs of 14 and 27 vertices, respectively. Previously, the lowest d for the symmetry level found was $\frac{n-5}{n}$ [5]. We have found a graph with the $\frac{n-7}{n}$ symmetry level on 27 vertices based on randomized constructions.

Based on the symmetric difference of the neighborhoods of the vertices, we show that $S(\Gamma)$ is bounded below by $\frac{1}{2}$ with increasing order of a graph. We recall the ideas of [1] who dealt with the measure of asymmetry, denoted $A(\Gamma)$, defined as the minimum number of edges that have to be added, $A^+(\Gamma)$, or deleted, $A^-(\Gamma)$, to obtain non-trivial automorphism. Both measures of (a)symmetry, $A(\Gamma)$ and $S(\Gamma)$, have the same general bound of $\frac{1}{2}$, suggesting a possible relation between the two. The bound is the same for asymmetric trees. However, there are graphs where $A^-(\Gamma) < S(\Gamma)$, and $S(\Gamma) < A^-(\Gamma)$.

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A LOWER BOUND
FOR THE COMPLEX FLOW NUMBER

Davide Mattiolo, Giuseppe Mazzuoccolo,
Jozef Rajník*, Gloria Tabarelli

A *complex nowhere-zero r-flow* on a graph G is a flow using complex numbers whose Euclidean norm lies in the interval $[1, r-1]$. The *complex flow number* of a bridgeless graph G, denoted by $\phi_{\mathbb{C}}(G)$, is the minimum of the real numbers r such that G admits a complex nowhere-zero r-flow.

The exact computation of $\phi_{\mathbb{C}}$ seems to be a hard task even for very small and symmetric graphs. In particular, the exact value of $\phi_{\mathbb{C}}$ is known only for families of graphs where a lower bound can be trivially proved. In this talk, we use geometric and combinatorial arguments to give a non-trivial lower bound for $\phi_{\mathbb{C}}(G)$ in terms of the odd-girth of a cubic graph G (i.e. the length of a shortest odd cycle) and we show that this lower bound is tight. Our main result relies on proving that the complex flow number of the wheel graph W_n of order $n+1$ is

$$\phi_{\mathbb{C}}(W_n) = \begin{cases} 2 & \text{if } n \text{ is even,} \\ 1 + 2\sin(\frac{\pi}{6} \cdot \frac{n}{n-1}) & \text{if } n \equiv 1, 3 \pmod 6, \\ 1 + 2\sin(\frac{\pi}{6} \cdot \frac{n+1}{n}) & \text{if } n \equiv 5 \pmod 6. \end{cases}$$

In particular, we show that for every odd n, the value of $\phi_{\mathbb{C}}(W_n)$ arises from one of three suitable configurations of points in the complex plane according to the congruence of n modulo 6.

The presenter of this talk acknowledges partial support from the research grants VEGA 1/0727/22, APVV-19-0308, VEGA 1/0743/21.

SPECTRAL PROPERTIES OF TOKEN GRAPHS

Mónica Reyes*, Cristina Dalfó,
Miquel Àngel Fiol

In graph theory, there are different operations that construct a "large" graph from a "smaller" one. Then, the question is what properties of the former can be deduced (or, at least, approximated) from the properties of the latter. One of these operations that recently received some attention in the literature is the construction of token graphs. The k-token graph $F_k(G)$ of a graph G is the graph whose vertices are the k-subsets of vertices from G, two of which being adjacent whenever their symmetric difference is a pair of adjacent vertices in G.

In this talk, we'll discuss some interesting facts about spectral properties of token graphs.

LIST STRONG EDGE-COLORING OF SUBCUBIC GRAPHS

Borut Lužar, Edita Máčajová,
Roman Soták*, Diana Švecová

A strong edge-coloring of a graph is an edge-coloring in which every color class is an induced matching. In the talk, we will consider the list version of the strong edge-coloring. Dai et al. (Strong list-chromatic index of subcubic graphs, Discrete Math., 341(12) (2018), 3434–3440) proved that every subcubic graph admits a strong list-chromatic index at most 11. We improve this result and give a tight upper bound for the strong list-chromatic index of subcubic graphs. In the proof, we use Combinatorial Nullstellesatz for reduction of small subgraphs and combine them to find a suitable strong edge-coloring for the shortest cycle with pendant edges.

We also discuss the relation between the strong chromatic index and the strong list-chromatic index of cubic graphs, and present an infinite class of cubic graphs where these two indices differ.

PARAMETERIZED ALGORITHMS FOR TRAVELING SALESPERSON PROBLEM AND ITS GENERALIZATIONS

Ondřej Suchý*

For many problems, the important instances from practice possess certain structure that one should reflect in the design of specific algorithms. Parameterized complexity offers a framework to design algorithms exploiting such a structure and also means to analyze how well particular algorithms scale as the input structure becomes more and more complex.

In this talk we survey some parameterized algorithms for the Traveling Salesperson Problem and its generalizations. We start with results concerning local search, where the parameter is the size of the solution neighborhood to be searched. Then we move on to the so called 'structural parameters' which measure the complexity of the structure of the input graph. The most prominent of such measures is the treewidth.

Finally, we focus on the kernelization results for the problem. Kernelization is a notion capturing efficient polynomial time data reductions, within the parameterized setting. Here, we present our recent results, e.g., a polynomial-size kernel with respect to vertex cover number or the size of a modulator to constant-sized components.

The talk is based on a joint work with Václav Blažej, Pratibha Choudhary, Jiong Guo, Sepp Hartung, Dušan Knop, Rolf Niedermeier, Šimon Schierreich, and Tomáš Valla.

ON THE NUMBER OF 4-REGULAR DISTANCE MAGIC CIRCULANTS

Tereza Škublová

A circulant $Circ(n, S)$ is a Cayley graph $Cay(G, S)$, where G is the finite cyclic group Z_n. We will suppose that $S \neq \emptyset$, $S = -S$ and $[S] = Z_n$. We call a circulant $G = Circ(n, S)$ distance magic if there exists a bijection ℓ from vertex set of G to the set $\{1, 2, \ldots, n\}$ such that for each vertex x the sum of values of function ℓ through the vertices adjacent to vertex x is constant for all vertices x of G.

Štefko Miklavič and Primož Šparl gave us the full classification of the 4-regular distance magic circulants in their article Classification of tetravalent distance magic circulant graphs, Discrete Mathematics, 2021. In this talk we will determine, for a given number of vertices n, the number of different 4-regular distance magic circulants.

IMPROPER VERTEX COLORING

Alexandra Kolačkovská, Mária Maceková,
Roman Soták, Diana Švecová*

Improper vertex coloring is such an assignment of colors to the vertices of a graph where adjacent vertices do not necessarily receive distinct colors. In addition, one can consider improper colorings with an allowed defect d_c assigned for every used color c (i.e., the maximum degree of the graph induced on the vertices of color c). While there are many results about such colorings for the class of planar graphs, not much is known about improper colorings of graphs embeddable on other surfaces. We study the problem of improper coloring for graphs embeddable on surfaces of Euler genus greater than 0. In the talk, we will give a brief overview of known results and present new contribution our research has brought, being particularly focused on improper coloring of toroidal graphs. Specifically, we will give upper bounds for color defects when coloring graphs with 4 colors, and prove tight results in terms of color defects when 5 and 6 colors are used.

THE MAX-BOND PROBLEM: SOME RESULTS

Hans Raj Tiwari

A cut in graph G is called a bond if each of the two parts of the cut induce connected subgraphs in G. Computing the maximum weight bond is an NP-hard problem even for planar graphs. $(K_5 - e)$ graphs present a tractable class of graphs over which this problem can be solved efficiently. A result of Wagner states that such graphs can be decomposed into k-clique-sums of the wheel graph and a few other constant sized graphs, for $k = 1, 2$. Using this decomposition and machinery from computational theories of bounded treewidth graphs, one can derive linear time algorithm for such graphs. We show how to remove this machinery by giving a simple algorithm that works for the wheel graph.

Furthermore, polyhedral charactrization of the band polytope is known only for the graphs in Wagner's decomposition theorem. We show how to obtain linear descriptions of the bond polytope of graphs that are k-sum (for $k = 1, 2$) of other graphs with known bond polytope. As a consquence of this we obtained that the bond problem admits linear size linear programs for $(K_5 - e)$-minor free graphs.

This is joint work with Petr Kolman.

HOFFMAN-SINGLETON GRAPH AND OVALS IN THE PROJECTIVE PLANE OF ORDER 5

Dávid Wilsch*, Martin Mačaj

The Hoffman-Singleton graph is the unique Moore graph with degree 7 and diameter 2. There is a long-standing open problem surrounding this graph. Can 7 of its copies be packed into the complete graph K_{50} such that they are edge-disjoint? In 2003, Šiagiová and Meszka used methods from topological graph theory to construct a set of five edge-disjoint copies of the Hoffman-Singleton graph in K_{50} which share a common group of automorphisms of order 25.

We completely classify all possible edge-disjoint quintuples of Hoffman-Singleton graphs that share such an automorphism group and show their correspondence to special sets of ovals in projective plane of order 5. We also search for similar sets of ovals in projective planes of higher orders.

LIST OF PARTICIPANTS CSGT 2024

June 3–7, 2024, Trojanovice, Czech Republic

Martin Bachratý

Slovak University of Technology in Bratislava

e-mail: `martin.bachraty@stuba.sk`

Jakub Balabán

Masaryk University

e-mail: `balaban.jakub@gmail.com`

Igor Balla

Masaryk University

e-mail: `iballa1990@gmail.com`

Peter Czimmermann

University of Žilina

e-mail: `peter.czimmermann@fri.uniza.sk`

Leonard Chidiebere Eze

Comenius University

e-mail: `leonard.eze@fmph.uniba.sk`

Igor Fabrici

P. J. Šafárik University in Košice

e-mail: `igor.fabrici@upjs.sk`

Andrea Feňovčíková

Technical University of Košice

e-mail: `andrea.fenovcikova@tuke.sk`

Dalibor Fronček

University of Minnesota Duluth

e-mail: `dalibor@d.umn.edu`

Babak Ghanbari

Charles University

e-mail: `babak@iuuk.mff.cuni.cz`

Štefánia Glevitzká

Comenius University

e-mail: `glevitzka1@uniba.sk`

Štefan Gyürki
Slovak University of Technology in Bratislava
e-mail: stefan.gyurki@stuba.sk

Róbert Jajcay
Comenius University
e-mail: robert.jajcay@fmph.uniba.sk

Jan Jedelský
Masaryk University
e-mail: xjedelsk@fi.muni.cz

Tomáš Kaiser
University of West Bohemia
e-mail: kaisert@kma.zcu.cz

František Kardoš
Comenius University
e-mail: fkardos@labri.fr

Tereza Klimošová
Charles University
e-mail: tereza@kam.mff.cuni.cz

Martin Knor
Slovak University of Technology in Bratislava
e-mail: knor@math.sk

Martin Kopřiva
University of West Bohemia
e-mail: mkopriva@kma.zcu.cz

Pavol Kollár
Comenius University
e-mail: pavol.kollar@fmph.uniba.sk

Petr Kovář
VSB – Technical University of Ostrava
e-mail: petr.kovar@vsb.cz

Tereza Kovářová
VSB – Technical University of Ostrava
e-mail: tereza.kovarova@vsb.cz

Jan Kratochvíl
Charles University
e-mail: honza@kam.mff.cuni.cz

Robert Lukoťka
Comenius University
e-mail: lukotka@dcs.fmph.uniba.sk

Tomáš Madaras
P. J. Šafárik University in Košice
e-mail: tomas.madaras@upjs.sk

Lukáš Málik
Masaryk University
e-mail: 514189@mail.muni.cz

Matúš Matok
Comenius University
e-mail: matus.matok@fmph.uniba.sk

Mariusz Meszka
AGH University of Science and Technology
e-mail: meszka@agh.edu.pl

Roman Nedela
University of West Bohemia
Matematický ústav SAV
e-mail: nedela@kma.zcu.cz

Makuochukwu Felix Oguagbaka
Comenius University
e-mail: felix.oguagbaka@fmph.uniba.sk

Ján Pastorek
Comenius University
e-mail: jan.pastorek@fmph.uniba.sk

Jozef Rajník
Comenius University
e-mail: jozef.rajnik@fmph.uniba.sk

Mónica Reyes
Comenius University
University of Lleida
e-mail: monicaandrea.reyes@udl.cat

Roman Soták
P. J. Šafárik University in Košice
e-mail: roman.sotak@upjs.sk

Ondřej Suchý
Czech Technical University in Prague
e-mail: ondrej.suchy@fit.cvut.cz

Jana Šiagiová
Slovak University of Technology in Bratislava
e-mail: jana.siagiova@stuba.sk

Tereza Škublová
Comenius University
e-mail: skublova3@uniba.sk

Diana Švecová
P. J. Šafárik University in Košice
e-mail: diana.svecova@gmail.com

Hans Raj Tiwary
Charles University
e-mail: hansraj@kam.mff.cuni.cz

Dávid Wilsch
Comenius University
e-mail: david.wilsch@fmph.uniba.sk

Yifan Zhang
VSB – Technical University of Ostrava
e-mail: yifan.zhang.st@vsb.cz