



EQUIANGULAR LINES AND REGULAR GRAPHS

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In 1973, Lemmens and Seidel posed the problem of determining the maximum number of equiangular lines in \mathbb{R}^r with angle $\arccos(\alpha)$ and gave a partial answer in the regime $r \leq 1/\alpha^2 - 2$. At the other extreme where r is at least exponential in $1/\alpha^2$, recent breakthroughs using graph-theoretic ideas have almost completely resolved this problem.

In this talk, we will discuss how orthogonal projection of matrices can be used in order to unify and improve upon all previous approaches, thereby yielding bounds which bridge the gap between the aforementioned regimes and are best possible either exactly or up to a small multiplicative constant. As a byproduct, we obtain the first extension of the celebrated Alon–Boppana theorem to dense graphs, with equality for strongly regular graphs corresponding to families of $\binom{r+1}{2}$ equiangular lines in \mathbb{R}^r .