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Department of Applied Mathematics
VŠB – Technical University Ostrava, 17. listopadu 2172/15, Ostrava–Poruba

SEMI-MAGIC SQUARES OVER DIHEDRAL GROUPS

Dalibor Fronček

Magic squares are among oldest known combinatorial objects, dating back to the 4-th century BC. A *magic square* of side n, $MS(n)$, is an $n \times n$ array with entries $1, 2, \ldots, n^2$, each appearing exactly once, such that every row, column, and the main forward and backward diagonals have the same sum $m = n(n^2+1)/2$, called the *magic constant*. A *semi-magic square* only requires the row and column sums to be equal.

Last year in Bardejovské Kúpele I presented results on dihedral supermagic labelings of some 4-regular graphs and observed that the results can be rather easily extended in certain classes of $4k$-regular graphs. Of the remaining regularities, the odd ones seem to be difficult, so I looked into $(4k+2)$-regular graphs (with no success so far).

Since supermagic labelings of regular complete bipartite graphs are equivalent to semi-magic squares, I got soon attracted by constructions of *dihedral semi-magic squares* $SMS_{D_k}(n)$, where the entries are elements of D_k rather than integers. Obviously, for odd n such squares do not exist because D_k is of order $2k$.

Not too surprisingly I was able to find such rectangles of side $n \equiv 0 \pmod 4$, but not $n \equiv 2 \pmod 4$. I will present some constructions of $SMS_{D_k}(n)$ where $n \equiv 0 \pmod 4$ and $n^2 = 2k$.