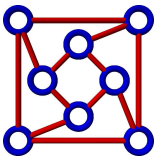

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EDGE- AND VERTEX-GIRTH-REGULAR GRAPHS AND THEIR ROLE IN EXTREMAL GRAPH THEORY

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The concepts of edge- and vertex-girth-regular graphs stem from two intersecting areas of interest: Edge- and vertex-transitive graphs and graphs extremal with regard to the well-known Degree/Diameter and Cage Problems.
We call a k-regular graph of girth g *edge-girth-regular* if each of its *edges* belongs to the same number λ of girth cycles, and we call a k-regular graph of girth g *vertex-girth-regular* if each of its *vertices* is included in the same number (denoted by λ again) of girth cycles. Clearly, every edge-transitive graph is edge-girth-regular (and also vertex-girth-regular) and each vertex-transitive graph is vertex-girth-regular. Moreover, all the Moore graphs are necessarily edge-girth-regular; thereby connecting the topic to the Extremal Graph Theory problems mentioned above. This suggests that the study of these two classes of graphs might potentially lead to useful insights in both of the above-mentioned areas of Graph Theory.
In our talk, we shall address a variety of questions concerning edge- and vertex-girth-regular graphs. First of all, for both classes, we derive upper bounds on the parameter λ viewed as a function of the degree and girth of the considered graphs. This naturally leads to existence problems in which we ask about the existence of graphs with permissible parameters (k, g, λ). With regard to this question, we will present infinite classes of admissible parameter triples for which we can show that graphs with these parameters do not exist, as well as a number of constructions in the other case. Ultimately, in the case when the existence question can be answered in positive, in line with the Cage Problem, we address the question of the minimal order of a (k, g, λ) edge- or vertex-girth-regular graph. We derive some lower bounds for these orders as functions of the parameters, and present some settled cases in which we know the exact answers (sometimes relying on the use of computers).