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$\mathcal{H}$-CLIQUE-WIDTH

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We generalize clique-width by defining $\mathcal{H}$-clique-width. We discuss its basic properties, its relation to planar product structure, and first-order transductions thereof.

We say that H is a *loop graph* if the vertex set of H is finite and H has no parallel edges. Let $H \in \mathcal{H}$ be a loop graph, and let $k \in \mathbb{N}$ be natural number. We say that ϕ is an (H, k)-expressing if ϕ uses the following operations to produces a graph $G = \phi()$ where every vertex $v \in V(G)$ is assigned a color $c_v \in [k]$ and a parameter vertex $p_v \in V(H)$:

- $\mathsf{create\_vertex}(c,\ p)$, creating a single vertex graph whose unique vertex has color $c \in [k]$ and parameter vertex $p \in V(H)$;
- $\mathsf{disjoint\_union}(\psi_1, \psi_2)$, creating a disjoint union of graphs $G_1 = \psi_1()$ and $G_2 = \psi_2()$
- $\mathsf{add\_edges}(\psi_1, c_1 \neq c_2)$, adding edges between all pairs of vertices u, v of $G_1 = \psi_1()$ satisfying all of the following conditions:
  - color of u is c_1
  - color of v is c_2
  - parameter vertices p_u of u and p_v of v are adjacent in H.
- $\mathsf{recolor}(\psi_1, c_1 \neq c_2)$, recoloring all vertices of $G_1 = \psi_1()$ whose color is c_1 to c_2 (without changing their parameter vertices).

Let $\mathcal{H}$ be a class of loop graphs. We say that a simple graph G has $\mathcal{H}$-*clique-width* at most k if there exists a loop graph $H \in \mathcal{H}$ and (H, k)-expression ϕ such that $G = \phi()$.