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## **THREE-CUTS ARE A CHARM: ACYCLICITY IN 3-CONNECTED CUBIC GRAPHS**

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In 2023, the three authors solved a conjecture (also known as the  $S_4$ -Conjecture) made by Mazzuoccolo in 2013: Let  $G$  be a bridgeless cubic graph. Then there exist two perfect matchings of  $G$  such that the complement of their union is a bipartite subgraph of  $G$ . This is a step closer to comprehend better the Fan–Raspaud Conjecture and eventually the Berge–Fulkerson Conjecture. The  $S_4$ -Conjecture, now a theorem, is also the weakest assertion in a series of three conjectures made by Mazzuoccolo in 2013, with the next stronger statement being: For every bridgeless cubic graph  $G$  there exist two perfect matchings of  $G$  such that the complement of their union is an acyclic subgraph of  $G$ . Unfortunately, this conjecture is not true: Jin, Steffen, and Mazzuoccolo later showed that there exists a counterexample admitting 2-cuts. Here we show that, despite of this, every cyclically 3-edge-connected cubic graph satisfies this second conjecture.