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COVERING COMPLETE GRAPHS BY K_3 AND K_4

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In a paper by D. Lukáš, P. Kovář, T. Kovářová, and M. Merta, "A parallel fast boundary element method using cyclic graph decompositions" from 2015 the authors proposed a method for decomposing calculations of a dense matrix into submatrices using a cyclic decomposition of complete graph K_n into n complete subgraphs K_k. The parameters n and k of the cyclic decomposition are not independent, since $n = k^2 - k + 1$. In addition, the existence of cyclic decomposition is based on the so-called ρ-labelings, the existence of which is known only if $k - 1$ is a power of a prime number. Also, using cyclic decompositions limits the approach to systems with shared memory.

Due to the development of supercomputer architecture, the possibility of implementation using fast local memories of individual cores is emerging, however the size of the local memory is limited. Therefore it makes sense to study decompositions of K_n into K_k, where k is a small fixed integer while n grows arbitrarily with the number of supercomputer cores. Such decompositions cannot be cyclic. The decompositions are based, for example, on Steiner triple systems or quadruple systems. Moreover, for many values of n, such decompositions do not exist.

On the other hand, instead of decompositions, graph coverings based on the constructions of combinatorial designs, can be successfully used. A small part of the calculation is duplicated. This can be modeled by duplicated edges, called excess. We show that for simultaneous covers by K_3 and K_4 exist for all values of n, and the size of the excess is constant and reasonably small. We answer almost completely the question of the existence of covers with the smallest possible number of covering graphs.