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Department of Applied Mathematics
VŠB – Technical University Ostrava, 17. listopadu 2172/15, Ostrava–Poruba

EDGE COLOURINGS OF CUBIC POLES

Robert Lukoťka[*]

Cubic graphs can be split into two classes: colorable graphs which admit a proper-3-edge colouring and uncolourable graphs which do not. Bridgeless cubic graphs that are not 3-edge-colourable, called snarks, are at the hearth of many important open problems like Cycle Double Cover Conjecture, Five Flow Conjecture, and Fulkerson Conjecture. To understand uncolourability of a snark one can take a cut containing k edges and split the graph into two graph parts, called cubic k-poles.

Each 3-edge-colouring of a k-pole induces a k-tuple of colours on the dangling edges, called boundary colourings. All colourings of the k-pole induce a (multi)set of boundary colourings, called colouring set. A colouring set contains only colourings satisfying parity lemma and the set has to be closed under Kempe switches. For $k \leq 5$ these two conditions are not only necessary but also sufficient. We will focus on the case where $k = 6$. We introduce a new equivalence relation that greatly reduces the number of colouring sets one needs to consider. We present the results of computational experiments using this equivalence relation.

If we restrict ourselves to planar graphs, then besides planarity requirement for Kempe switches, we have another tool that restricts colouring sets of k-poles, Four Colour Theorem. As an possible additional tool we will use flow polynomials. We show that certain generalised flow polynomials are an efficient tool to capture the number of colourings with given boundary. We explore conditions that Four Colour Theorem imposes on the polynomial and study k-poles that are close to “refuting” the Four Colour Theorem with respect to their polynomial coefficients.