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ON ANTI-A-WALKS IN PLANE GRAPHS

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Given a graph G and its rotation system $\mathcal{R} = \bigcup_{v \in V(G)} R(v)$ (where $R(v)$ is a cyclic clockwise ordering of edges incident with v), a walk $(v_0, e_1, v_1, \dots, e_k, v_k)$ in G is an *anti-A-walk* if, for every $i \in \{1, \dots, k-1\}$, the edges e_i, e_{i+1} are not successive in $R(v_i)$. A particular example of such a walk is earlier known *cut-through trail* in a 4-regular plane graph (where the edges e_i, e_{i+1} are opposite in $R(v_i)$).

Focusing on study of properties of anti-A-walks in plane graphs, we address the question of their existence between each pair of vertices. While this is not always possible in 4-regular plane graphs (resp. plane graphs of minimum degree at least 4) in the sense of existence of cut-through paths, the problem is open (even with anti-A-walks) for plane graphs of minimum degree 5. We developed a set of tools (in Maple computer algebra system) for testing path anti-A-connectivity of 5-regular plane graphs; in addition, for finding the shortest anti-A-paths (resp. trails) between two vertices in a graph with given rotation system, we used integer linear programming approach. We also present large families of polyhedral graphs of minimum degree 5 (with exponentially many members for fixed number of vertices) in which every pair of vertices is connected by an anti-A-path.