TWO-FACTORIZATIONS OF SOME REGULAR GRAPHS

Mariusz Meszka

A *k-factor* in a graph G is a k-regular spanning subgraph of G. A *k-factorization* of G is a collection $\{F_1, F_2, \ldots, F_t\}$ of edge-disjoint k-factors such each edge of G belongs to exactly one F_i. We say that G has an *F-factorization* if each F_i, $i = 1, 2, \ldots, t$, is isomorphic to F.

One of the best-known open problems concerning two-factorizations is the famous Oberwolfach problem, posed by G. Ringel in 1967, which asks whether, for any two-factor F, the complete graph K_n (when n is odd) or $K_n \setminus I$ (when n is even and I is a one-factor removed from K_n) admits an F-factorization. Several years later A. Rosa suggested the following extension of the Oberwolfach problem, the so-called Hamilton–Waterloo problem, which asks for the existence of a two-factorization of K_n or $K_n \setminus I$ (depending on the parity of n) in which r of its two-factors are isomorphic to a given two-factor R, and the remaining q two-factors are isomorphic to a given two-factor Q, for any admissible r and q.

Results related to both these problems will be widely discussed. Moreover, various algorithmic methods for constructing two-factorizations, together with their relationship to other combinatorial objects and applications, will be presented.