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COHERENT PARTITIONS OF CUBIC GRAPHS

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A partition $\{A, J\}$ of the vertex-set of a connected cubic graph G with n vertices is *coherent* if A induces a tree, and J induces a graph with at most one edge. By parity argument, J is independent if $n \equiv 2 \pmod 4$, and J induces a one-edge graph if $n \equiv 0 \pmod 4$. In 1975 Payan and Sakarovitch proved that a cyclically 4-connected cubic graph with $n \equiv 2 \pmod 4$ admits a coherent partition. In 2024 (Discrete Mathematics) we confirmed the existence of coherent partition for cyclically 4-connected cubic graph with $n \equiv 0 \pmod 4$, and for some well-defined families of cubic graphs of cyclic connectivity 3. Motivation for investigation of coherent partitions in cubic graphs is 3-fold. First, if G is admits a coherent partition $\{A, J\}$, then the general lower bound $\Phi(G) \geq \lceil \frac{n+2}{4} \rceil$ for the decycling number (feedback number) $\Phi(G)$ is achieved. Secondly, we proved that a cubic graph G which admits a coherent partition is upper-embeddable, and we have a conjecture that upper-embeddability of G is equivalent to existence of a coherent partition of G. Hence there is a close relation between coherent partitions and maximum genus of cubic graph. Finally, coherent partitions appear as a useful tool in constructions of Hamilton cycles, or Hamilton paths in cubic graphs belonging to certain important families of cubic graphs drawn on surfaces (including leapfrog fullerines or cubic Cayley graphs).

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