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# CUBIC GRAPHS THAT ARE FAR FROM BEING COVERABLE BY FOUR PERFECT MATCHINGS

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Cubic graphs without bridges can be categorized in terms of their perfect matching index—the minimum number of perfect matchings needed to cover the edges of a bridgeless cubic graph. A conjecture of Berge suggests that, at most, five perfect matchings are needed to cover any such graph. Cubic graphs that need a minimum of five perfect matchings to cover their edges are very rare, and only a few infinite families of nontrivial examples of such graphs exist. Such graphs are of particular interest, as famous, longstanding open problems, including the cycle double cover conjecture, Tutte's 5-flow conjecture, and Berge–Fulkerson conjecture, can be reduced to them.
In our talk, we define the four perfect matching cover defect of a cubic graph as the minimum number of edges of the graph not covered by four perfect matchings. This parameter serves as a way to capture how far a cubic graph is from being coverable by four perfect matchings. We also introduce several other invariants that capture this notion. Additionally, we construct an infinite family of cubic graphs that are cyclically 4-edge-connected, have girth at least 5, and are far from being coverable by four perfect matchings with respect to the invariants. Our study shows that each member of the infinite family has the same value for each invariant. However, to examine the differences between the invariants, we present a cubic graph that differs from the members of the infinite family in that it has two different values across the invariants.

**keywords:** cubic graph, snark, flow, covering, cyclic connectivity, perfect matching

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