CSGT 2024, June 3–7 2024, Trojanovice
59th Czech-Slovak Conference on Graph Theory 2024
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ASYMMETRIC GRAPHS AND PARTIAL AUTOMORPHISMS

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Most of the graphs are *asymmetric*, i.e., they only have a trivial automorphism group [1]. However, the relation between asymmetric graphs, regular graphs, and graphs with non-trivial automorphism group is challenging to delineate. Removing just a single node from the graphical regular representation yields an asymmetric graph [2]. There are minimal asymmetric graphs, where all vertex-induced graphs have non-trivial automorphisms [3]. Moreover, almost all regular graphs have only a trivial automorphism group [4], one of the smallest being the well-known Frucht graph.

Following up on the research of Jajcayova et al. [5], we investigate the symmetry level of the graphs. *Symmetry level* of a graph Γ, $S(\Gamma)$, is defined as the ratio between the largest rank of a non-trivial partial automorphism of a graph Γ and its order $|V(\Gamma)|$, that is, $S(\Gamma) = \frac{n-d}{n}$.

Currently, we have graphs between $\frac{3}{4}$ and $\frac{2}{3}$ symmetry levels on graphs of 14 and 27 vertices, respectively. Previously, the lowest d for the symmetry level found was $\frac{n-5}{n}$ [5]. We have found a graph with the $\frac{n-7}{n}$ symmetry level on 27 vertices based on randomized constructions.

Based on the symmetric difference of the neighborhoods of the vertices, we show that $S(\Gamma)$ is bounded below by $\frac{1}{2}$ with increasing order of a graph. We recall the ideas of [1] who dealt with the measure of asymmetry, denoted $A(\Gamma)$, defined as the minimum number of edges that have to be added, $A^{+}(\Gamma)$, or deleted, $A^{-}(\Gamma)$, to obtain non-trivial automorphism. Both measures of (a)symmetry, $A(\Gamma)$ and $S(\Gamma)$, have the same general bound of $\frac{1}{2}$, suggesting a possible relation between the two. The bound is the same for asymmetric trees. However, there are graphs where $A^{-}(\Gamma) < S(\Gamma)$, and $S(\Gamma) < A^{-}(\Gamma)$.

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