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A LOWER BOUND FOR THE COMPLEX FLOW NUMBER

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A *complex nowhere-zero r-flow* on a graph G is a flow using complex numbers whose Euclidean norm lies in the interval $[1, r-1]$. The *complex flow number* of a bridgeless graph G, denoted by $\phi_{\mathbb{C}}(G)$, is the minimum of the real numbers r such that G admits a complex nowhere-zero r-flow.

The exact computation of $\phi_{\mathbb{C}}$ seems to be a hard task even for very small and symmetric graphs. In particular, the exact value of $\phi_{\mathbb{C}}$ is known only for families of graphs where a lower bound can be trivially proved. In this talk, we use geometric and combinatorial arguments to give a non-trivial lower bound for $\phi_{\mathbb{C}}(G)$ in terms of the odd-girth of a cubic graph G (i.e. the length of a shortest odd cycle) and we show that this lower bound is tight. Our main result relies on proving that the complex flow number of the wheel graph W_n of order $n+1$ is

$$\phi_{\mathbb{C}}(W_n) = \begin{cases} 2 & \text{if } n \text{ is even,} \\ 1 + 2\sin(\frac{\pi}{6} \cdot \frac{n}{n-1}) & \text{if } n \equiv 1, 3 \pmod 6, \\ 1 + 2\sin(\frac{\pi}{6} \cdot \frac{n+1}{n}) & \text{if } n \equiv 5 \pmod 6. \end{cases}$$

In particular, we show that for every odd n, the value of $\phi_{\mathbb{C}}(W_n)$ arises from one of three suitable configurations of points in the complex plane according to the congruence of n modulo 6.

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