Invited lecture

HOMOMORPHISMS WITH LOCAL CONSTRAINTS — STRUCTURE AND COMPLEXITY

Jiří Fiala

A graph homomorphism $f : G \to H$ is locally bijective if for every vertex u of G the mapping f acts bijectively between the neighborhood of u and the neighborhood of $f(u)$. We define locally injective and locally surjective homomorphisms analogously.

A degree partition of a graph is an equivalence on its vertices such that equivalent vertices cannot be distinguished by counting their neighbors in any equivalence class. A square matrix that quantitatively describes adjacencies between the classes of a degree partition is called degree matrix.

We express the existence of a locally constrained homomorphism between two graphs as two kinds of binary relations: one on graphs, and another on degree matrices.

These structures have interesting combinatorial properties, e.g. they impose a partial order. In the talk we review some classical as well as some recent results on this subject. We apply them in a study of the computational complexity of the related problems.