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THE LOEBL-KOMLOS-SOS CONJECTURE

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Extremal graph theory investigates which general properties of a graph force the appearance of a given substructure. For illustration, I mention Mantel's theorem, which states that any n-vertex graph with more than $\lceil \frac{n}{2} \rceil \cdot \lfloor \frac{n}{2} \rfloor$ edges contains a triangle. In this talk, I will the following setting. Given a family of trees, what general properties of a graph ensure the presence of the copy of every member of this family? If we consider the family of all trees of order k, a minimum degree of $k-1$ is sufficient, as a simple greedy algorithm finds the desired copy. There are two ways to relax the condition of the minimum degree. If we consider the average degree instead, this leads to the famous Erdős–Sós Conjecture, which asserts that any graph with average degree strictly larger than $k-2$ contains a copy of any tree of order k. On the other hand, one can consider the median degree instead. A graph G has *median degree* d, if half of its vertices have degree at least d and the other half have degree at most d.

Conjecture 1 (Loebl-Komlós–Sós) *Every graph with median degree at least $k-1$ contains a copy of any tree of order k.*

I shall present the solution of the Loebl–Komlós–Sós Conjecture for dense graphs (i.e., for graphs with $\Theta(n^2)$ edges) and the approximate solution of the conjecture for sparse graphs (i.e., for graphs with $o(n^2)$ edges). The tools used in the dense setting is a combination of Szemerédi's Regularity Lemma and the stability method. In the setting of sparse graphs, one cannot use directly Szemerédi's Regularity Lemma, as its assertion is void for such graphs. There exists a regularity lemma for sparse graphs, but its statement is not sufficiently general to approach this conjecture. To prove the approximate version of the Loebl–Komlós–Sós Conjecture, we used a novel graph decomposition technique that generalises Szemerédi's Regularity Lemma and is applicable to any graph with average degree at least c for some large constant $c \in \mathbb{N}$. This is joint work with Jan Hladký, János Komlós, Miklós Simonovits, Maya Stein, and Endre Szemerédi.