Invited lecture

K_t MINORS IN LARGE t-CONNECTED GRAPHS

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A graph G has a K_t minor if a graph isomorphic to K_t, the complete graph on t vertices, can be obtained from a subgraph of G by contracting edges. A long-standing conjecture of Hadwiger states that every graph with no K_t minor is $(t-1)$-colorable. Hadwiger's conjecture is known for $t \leq 6$, and open for all $t > 7$.

A deep theorem of Robertson and Seymour describes the structure of graphs with no K_t minor. The theorem is very powerful, but it is fairly complicated to state, and the condition it gives is necessary, but not sufficient, for the exclusion of a K_t minor.

We prove a necessary and sufficient condition under additional restrictions on the graph G. We prove that for every integer t there exists an integer N such that every t-connected graph on at least N vertices with no K_t minor has a set of at most $t-5$ vertices whose deletion makes the graph planar. This is best possible in the sense that neither t-connectivity nor the size of the deleted set can be lowered, and for $t > 7$ some lower bound on the number of vertices is needed.

This is joint work with Sergey Norin.